SU(5) Grand Unified Model and Dark Matter

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Abstract

A dark matter model which is called w-matter or mirror dark matter is concretely constructed based on (f-SU(5))X(w-SU(5)) symmetry. There is no Higgs field and all masses originate from interactions in the present model. W-matter is dark matter relatively to f-matter and vice versa. In high-energy processes or when temperature is very high, visible matter and dark matter can transform from one into another. In low-energy processes or when temperature is low, there is only gravitation interaction of dark matter for visible matter.

Keywords: Dark matter; origin of mass.

1. Lagrangian of the $SU_f(5) \times SU_w(5)$ model

Conjecture 1 *There are two sorts of matter which are called* $fire-matter$ $(f-matter)$ *and* $water-matter$ $(w-matter)$*, respectively. Both are symmetric and have* $SU_f(5) \times SU_w(5)$ *symmetry. There is no other interaction except the gravitation between both and the coupling* (5) *of f-scalar fields and w-scalar fields.*

The conjecture, in fact, is a necessary inference of a quantum field theory without divergence in which all loop-corrections are finite and the energy density ρ_0 of the vacuum state must be zero without normal ordering of operators[3]. It is obvious that the conjecture is consistent with a sort of dark matter model which is called $w-matter$[3] or mirror dark matter[4].

Based the conjecture, the Lagrangian density of the $SU_f(5)\times SU_w(5)$ model can be taken as

$$\mathcal{L}=\mathcal{L}_f\left(\chi_f,\Psi_f,G_f,\Phi_f,H_f\right)+\mathcal{L}_w\left(\chi_w,\Psi_w,G_w,\Phi_w,H_w\right)+\mathcal{L}_\Omega+V, \tag{1}$$

$$V=V_f+V_w+V_\Omega+V_I,$$

$$V_f=\frac{1}{4}a\left(Tr\Phi_f^2\right)^2+\frac{1}{2}bTr\left(\Phi_f^4\right)+\frac{1}{4}\xi\left(H_f^+H_f\right)^2+\frac{1}{2}\varsigma H_f^+H_fTr\Phi_f^2-\frac{1}{2}\varkappa H_f^+\Phi_f^2H_f, \tag{2}$$

$$V_w=\frac{1}{4}a\left(Tr\Phi_w^2\right)^2+\frac{1}{2}bTr\left(\Phi_w^4\right)+\frac{1}{4}\xi\left(H_w^+H_w\right)^2+\frac{1}{2}\varsigma H_w^+H_wTr\Phi_w^2-\frac{1}{2}\varkappa H_w^+\Phi_w^2H_w,, \tag{3}$$

$$V_\Omega=\frac{1}{4}\lambda\Omega^4,\quad \mathcal{L}_\Omega=\frac{1}{2}\partial_\mu\Omega\partial^\mu\Omega, \tag{4}$$

$$V_I=-\frac{1}{15}w\Omega^2\left(Tr\Phi_f^2+Tr\Phi_w^2\right)-\frac{2A}{225}Tr\Phi_f^2Tr\Phi_w^2, \tag{5}$$

where χ and Ψ denote fermion fields, and G the $SU(5)$ gauge fields. Ω, Φ and H are the $\underline{1}$, $\underline{24}$ and $\underline{5}$ representations, respectively. It should be pointed out that all the scalar fields are not Higgs fields because they are all massless before symmetry breaking.

Similarly to the conventional $SU(5)$ model, the possible fermion states for the first generation are

$$\Psi_{fL}=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u_{f3}^c & -u_{f2}^c & -u_{f1} & -d_{f1} \\ -u_{f3}^c & 0 & u_{f1}^c & -u_{f2} & -d_{f2} \\ u_{f2}^c & -u_{f2}^c & 0 & -u_{f3} & -d_{f3} \\ u_{f1} & u_{f2} & u_{f3} & 0 & -e_f^+ \\ d_{f1} & d_{f2} & d_{f3} & e_f^+ & 0 \end{pmatrix}_L,\quad \Psi_{fR}=\begin{pmatrix} d_{f1} \\ d_{f1} \\ d_{f1} \\ e_f^+ \\ -\nu_{fe}^c \end{pmatrix}_R \tag{6}$$

$$\Psi_{wR}=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u^c_{w3} & -u^c_{w2} & -u_{w1} & -d_{w1} \\ -u^c_{w3} & 0 & u^c_{w1} & -u_{w2} & -d_{w2} \\ u^c_{w2} & -u^c_{w2} & 0 & -u_{w3} & -d_{w3} \\ u_{w1} & u_{w2} & u_{w3} & 0 & -e^+_w \\ d_{w1} & d_{w2} & d_{w3} & e^+_w & 0 \end{pmatrix}_R, \quad \Psi_{wL}=\begin{pmatrix} d_{w1} \\ d_{w1} \\ d_{w1} \\ e^+_w \\ -\nu^c_{we} \end{pmatrix}_L \tag{7}$$

The other possible model is an $SU(5)$ grand unified model with hadrons as nontopological solitons[2]. The conclusions of the present paper are independent of a concrete model.

2. Symmetry spontaneously breaking and temperature effects

For simplicity, we do not consider the couplings Ω and Φ with χ for a time. Ignoring the contributions of the scalar fields and the fermion fields to one loop correction and only considering the contribution of the gauge fields to one-loop correction, when $\overline{\varphi}_s \ll kT$, k is the Boltzmann constant, similarly to Ref. [1], the finite-temperature effective potential approximate to 1-loop in flat space can be obtained

$$\begin{aligned} V = {} & \frac{\lambda}{8}T^2\Omega^2+\frac{1}{4}\lambda\Omega^4-\frac{A}{2}\varphi_f^2\varphi_w^2-\frac{1}{2}w\Omega^2\left(\varphi_f^2+\varphi_w^2\right) \\ & +\frac{D}{4!}\varphi_f^4+B\varphi_f^4\left(\ln\frac{\varphi_f^2}{\sigma^2}-\frac{1}{2}\right)+CT^2\varphi_f^2 \\ & +\frac{D}{4!}\varphi_w^4+B\varphi_w^4\left(\ln\frac{\varphi_w^2}{\sigma^2}-\frac{1}{2}\right)+CT^2\varphi_w^2, \end{aligned} \tag{8}$$

where

$$\Phi_s = Diagonal\left(1,1,1,-\frac{3}{2},-\frac{3}{2}\right)\overline{\varphi}_s, \tag{9}$$

$$B\equiv\frac{5625}{1024\pi^2}g^4,\quad \frac{(225a+105b)}{16}\equiv\frac{D}{4!}+\frac{11}{3}B,\quad C\equiv\frac{75}{16}\left(kg\right)^2,$$

σ is regarded as a constant, and the terms independent of Ω and Φ are neglected.

According to the mirror dark matter model, the temperature of mirror matter is strikingly lower than that of visible matter. But this is not necessary when a cosmological model is considered. We will discuss the problem in another paper. The temperature T_f of $f-matter$ may be different from T_w of $w-matter$ in the present model as well, but for simplicity we take $T_f = T_w$.

The conditions by which V takes its extreme values are

$$\left[\lambda\overline{\Omega}^2 - w\left(\overline{\varphi}_f^2 + \overline{\varphi}_w^2\right) + \frac{\lambda}{4}T^2\right]\overline{\Omega} = 0, \tag{10a}$$

$$-w\overline{\Omega}^2 - A\overline{\varphi}_w^2 + \frac{D}{6}\overline{\varphi}_f^2 + 4B\overline{\varphi}_f^2 \ln\frac{\overline{\varphi}_f^2}{\sigma^2} + 2CT^2 = 0, \tag{10b}$$

$$-w\overline{\Omega}^2 - A\overline{\varphi}_f^2 + \frac{D}{6}\overline{\varphi}_w^2 + 4B\overline{\varphi}_w^2 \ln\frac{\overline{\varphi}_w^2}{\sigma^2} + 2CT^2 = 0. \tag{10c}$$

When $T \sim 0$,

$$\overline{\varphi}_f^2 = \overline{\varphi}_w^2 \equiv \sigma_0^2 = \sigma^2 \exp M, \quad M \equiv \frac{1}{4B}\left(A + \frac{2w^2}{\lambda} - \frac{D}{6}\right),$$

$$\overline{\Omega}_0^2 = \upsilon_0^2 = \frac{2w}{\lambda}\sigma^2 \exp M, \tag{11a}$$

$$V = V_{\min} = -B\sigma^4 \exp 2M. \tag{11b}$$

$\sigma^2(T)$ and $\upsilon^2(T)$ will decrease and $V_{\min}$ will increase as temperature rises. There must be the critical temperature T_{cr} so that when $T > T_{cr}$, the least value of V is $V\left(\overline{\varphi}_f = \overline{\varphi}_w = \overline{\Omega} = 0\right) = 0$. T_{cr} is rough estimated to be

$$T_{cr} = \frac{8B}{w + 8C}\sigma^2 \exp\left(M - \frac{1}{2}\right). \tag{12}$$

Ω is not absolutely necessary for the symmetry breaking of the present model, but it is necessary for some a cosmological model[5].

After spontaneous symmetry-breaking, the reserved symmetry is $[SU_f(3) \times SU_f(2) \times U_f(1)] \times [SU_w(3) \times SU_w(2) \times U_w(1)]$. The breaking is a sort of dynamical breaking. In other words, the interactions of the scalar fields with the gauge fields make the massless scalar fields become 'Higgs fields', and finally cause the spontaneous symmetry-breaking. As a consequence, the $f-particles$ ($w-particles$) can get their masses determined by the reserved symmetry $SU(3)\times SU(2)\times U(1)$ as the conventional $SU(5)$ GUT theory in which there are Higgs fields.

3. The physical significance of the present model

1. The model implies that symmtry breaking originates from interaction, and all masses originate from interactions.

2. $W-matter$ is dark matter for $f-matter$ in low energy process, vice versa. This is because the masses of the scalar particles to be very large in low temperature so that the transformation of the $f-$ and the $w-scalar$ particles from one into another and their effects may be ignored and there is no interaction except the coupling (5) and the gravitation between $f-matter$ and $w-matter$. This sort of dark matter is called mirror dark matter in Refs.[4].

3. In high-energy processes or when temperature is very high, visible matter and dark matter can transform from one into another. In such process energy seems to be non-conservational, because dark matter cannot be detected. The following reaction originating from (1) and (5) is an example in which visible matter transforms into dark matter.

$$p+\overline{p}\longrightarrow\varphi_{fA}\longrightarrow\varphi_{fB}+\varphi_{wC}+\varphi_{wD}. \tag{13}$$

In the reaction φ_{wC} and φ_{wD} and the $w-particles$ coming from the decay of φ_{wC} and φ_{wD} cannot be detected.

4. Conclusion

A dark matter model which is called $w - matter$ or mirror dark matter is concretely constructed based on $SU_f(5) \times SU_w(5)$ symmetry. There is no Higgs field and all masses originate from interactions in the present model. $W - matter$ is dark matter relatively to $f - matter$ and vice versa. In high-energy processes or when temperature is very high, visible matter and dark matter can transform from one into another. In such process energy seems to be non-conservational, because dark matter cannot be detected. In low-energy processes or when temperature is low, there is only gravitation interaction of dark matter for visible matter.

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