Non-Abelian Effects on Wake Potential in Quark-gluon Plasma

Jiang Bing-feng


IOPP       CCNU

jiangbf@iopp.ccnu.edu.cn
Outline

• Introduction

• Screening potential: a static and a moving charges

• Screening Potential induced by a fast parton → the Hard Thermal Loop resummation technique

• Conclusion
Introduction

Jet quenching → a possible signature for the QGP formation → radiative energy loss.

The experimental dihadron correlation function shows a double peak structure in the away side. (PRL95(2005)152301; PRL97(2006)052301)

Then, another aspects of the in-medium jet physics, wakes(trail effects) induced by jet, become hot topics.

Mach cone, Cherenkov radiation, large angle gluon radiation...... see a review: hep-ph/0701257.

induced current, induced charges, wake potential
Wake potential in the linear response theory

A fast parton $\rightarrow$ a constant velocity and a fixed moving direction, the external(test) charge density

$$
\rho^{a}_{\text{ext}} = 2\pi Q^{a}\delta(\omega - v \cdot k)
$$  \hspace{1cm} (1)

The induced color charge density by an external charge

$$
\rho^{a}_{\text{ind}}(\omega, k) = \left( \frac{1}{\epsilon_{L}(\omega, k)} - 1 \right) \rho^{a}_{\text{ext}}(\omega, k)
$$  \hspace{1cm} (2)

According to Poisson equation, the waked potential

$$
\Phi^{a}(\omega, k) = 4\pi \frac{\rho^{a}_{\text{ext}}(\omega, k)}{k^{2}\epsilon_{L}(\omega, k)}
$$  \hspace{1cm} (3)

$\epsilon_{L}$ is the dielectric function.
\[
\Phi^a(r, v, t) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \exp[i(k \cdot r - \omega t)] \cdot \frac{4\pi}{k^2 \epsilon_L(\omega, k)} \cdot 2\pi Q^a \delta(\omega - k \cdot v),
\]

The key point \(\rightarrow\) \(\epsilon_L(\omega = \vec{v} \cdot \vec{k}, k)\) \(\rightarrow\) the HTL approximation:

\[
\epsilon_{htl}^L(\omega, k) = 1 + \frac{m_D^2}{k^2} \left[1 - \frac{\omega}{2k} \left(\ln \left| \frac{\omega + k}{\omega - k} \right| - i\pi \Theta(k^2 - \omega^2)\right)\right],
\]

\(\nu = 0 \rightarrow \omega = 0\) the static case, \(\Phi^a(r, \nu, t) = \frac{Q \exp[-m_D r]}{r}\)

Yukawa potential

\(\nu \neq 0,\) \(\text{PLB 618}(2005)123,\) \(\text{PRD 74}(2006)094002.\)
\[ \epsilon_L(\omega = \vec{v} \cdot \vec{k}, k) \rightarrow \text{the HTL result is incomplete} \]

The higher loop contribution \( \rightarrow \) neglected.

In the HTL approximation, \( \epsilon_L(\omega = \vec{v} \cdot \vec{k}, k) \) of the QGP is similar to the result of the QED abelian plasma, \( m_r \rightarrow m_D \)

Except the color factors in \( m_D \), no more non-Abelian effects are involved in the dielectric function.

With the HTL resummation approach, \( \rightarrow \) the longitudinal dielectric function \( \rightarrow \) the wakepotential of the moving test charge.
The resummed dielectric function and the wake potential

- In Coulomb gauge

\[ \epsilon_L(\omega, k) = 1 - \frac{\Pi_{00}(K)}{k^2} \]

- The gluon self-energy

![Diagram showing one-loop gluon self-energy](image)

Fig 2. One-loop gluon self-energy

- In the HTL approximation

\[
\Pi_{L}^{\text{htl}}(K) = \Pi_{00}^{\text{htl}}(K) = -m_D^2 \left[ 1 - \frac{\omega}{2k} \ln\left(\frac{\omega+k}{\omega-k}\right) \right],
\]

\[
\Pi_{T}^{\text{htl}}(K) = \frac{m_D^2 \omega^2}{2} \left[ 1 - \left(1 - \frac{\omega^2}{k^2}\right) \frac{\omega}{2k} \ln\left(\frac{\omega+k}{\omega-k}\right) \right].
\]
In the HTL resummation approximation

• The integrals over loop momentom $\rightarrow$ “hard” ($\sim T$) and “soft” ($\sim gT$) momentom. Braaten & Pisarsiki, NPB 337&339, PRL 63&64.

• Results from integral over the hard momentum at high temperature $\Pi_{\mu\nu}^h(P) \rightarrow$ the HTL self-energy $\Pi_{\mu\nu}^{htl}(P)$

• In the case of the integrals over soft momentum, effective propagators and effective vertices are involved and the calculation are much complicated.
• Assuming the external gluon line is hard. The soft momentum contribution mainly comes from diagrams

![Diagrams](image)

**Fig 3. The effective gluon self-energy**

• The self-polarization of gluon $\Rightarrow$ no counterpart in Abelian plasma $\Rightarrow$ **non-Abelian characteristic of the QGP**

• The dotted line $\Rightarrow$ the effective gluon propagator $D^{*\mu\nu}$ is defined by the Schwinger-Dyson equation in the Coulomb gauge. Its longitudinal and transverse components are

$$D_{00}(K) = \Delta_L(K), \quad \Delta_L(K) = \frac{i}{k^2};$$

$$D_{0i}(K) = D_{i0}(K) = 0;$$

$$D_{ij}(K) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Delta_T(K), \quad \Delta_T(K) = \frac{i}{K^2}.$$
\[ \Delta_L^*(K) = \frac{i}{k^2 - \Pi_L^{htl}(K)}, \quad \Delta_T^*(K) = \frac{i}{K^2 - \Pi_T^{htl}(K)}. \]

- According to the effective gluon propagator, the effective gluon self-energy

\[
\Pi_{\mu \nu}^{*1}(K) = \frac{1}{2} i \int \frac{d^4 P}{(2\pi)^4} \Gamma_{\sigma \mu \rho}(-K - P, K, P) D_{\rho \rho'}^*(P) \\
\times \Gamma_{\rho' \nu \sigma}(-P, -K, P + K) D_{\sigma \sigma'}(P + K)
\]

\[
\Pi_{\mu \nu}^{*2}(K) = \frac{1}{2} i \int \frac{d^4 P}{(2\pi)^4} \Gamma_{\mu \nu \rho \sigma}(K, -K, P, -P) D_{\rho \sigma}^*(P)
\]

\[
\Pi_{\mu \nu}(K) = \Pi_{\mu \nu}^{*1}(K) + \Pi_{\mu \nu}^{*2}(K)
\]

\[
\Pi_{00}(K) = \Pi_{00}^{htl}(K) + \Pi_{00}^*(K), \quad \epsilon_L(\omega, k) = 1 - \frac{\Pi_{00}(K)}{k^2}
\]

- The resummed gluon self-energy and dielectric function
• Assuming the fast parton $\rightarrow z$, in the cylindrical coordination

$$k = (\kappa \cos \phi, \kappa \sin \phi, k_z), \quad r = (\rho, 0, z)$$

$$\Phi^a(\rho, z, t) = \frac{Q^a}{\pi v} \int_0^\infty d\kappa \kappa J_0(\kappa \rho) \int_{-\infty}^{\infty} d\omega \frac{1}{k^2 \Delta(\omega, k)}$$

$$\times \left[ \cos \left\{ \omega \left( \frac{z}{v} - t \right) \right\} \cdot \text{Re} \epsilon_L + \sin \left\{ \omega \left( \frac{z}{v} - t \right) \right\} \cdot \text{Im} \epsilon_L \right].$$

where $J_0$ is the Bessel function, $k = \sqrt{k^2 + \omega^2/v^2}$ and $\Delta = (\text{Re} \epsilon_L)^2 + (\text{Im} \epsilon_L)^2$

• Focusing on the screening potential parallel to the moving direction of the fast parton, $r \parallel v$ and $\rho = 0$

$$\Phi_{\parallel}^a(z, v, t) = \frac{1}{\pi} \int_0^\infty dk \int_{-1}^{1} dx \left\{ \frac{\text{Re} \epsilon_L}{\Delta} \cos(kx |z - vt| m_D) + \frac{\text{Im} \epsilon_L}{\Delta} \sin(kx |z - vt| m_D) \right\}$$

$$= \Phi_1 + \Phi_2.$$
Numerical results and discussion

- In numerical calculation, $g = 0.1$, $N_f = 2$, $C_A = 3$
- For comparison, HTL & HTL resummation

![Graph showing numerical results](attachment:image.png)
• Screening potential in forward-backward direction is anisotropic.
  • In the forward direction → Yukawa-like potential
  • In the backward direction → speed-dependent,
    \[ \nu = 0.55c \] Lennard-Jones-like potential with a negative minimum → a short repulsive and a long range attractive interaction.
    \[ \nu = 0.99c \] → Showing an obvious oscillation

For comparision of results → HTL and HTL resummation
• Resummation calculation enhances anisotropy of the screening potential.
  • In the forward direction → screening reduced in resummation
  • In the backward direction → the negative minimum → deeper

How to understand these properties?
The HTL dielectric function

\[
\frac{\Phi_{||}(z, v, t)}{Q^a m_D} = \frac{1}{\pi} \int_{-1}^{1} dx \int_{0}^{\infty} dk \left\{ \frac{\text{Re}\epsilon_L}{\Delta} \cos(kx|z - vt|m_D) + \frac{\text{Im}\epsilon_L}{\Delta} \sin(kx|z - vt|m_D) \right\}
\]

\[= \Phi_1 + \Phi_2.\]

- Symmetric properties about \(\omega\)

\[
\text{Re}\epsilon^{htl}_L(\omega, k) = \text{Re}\epsilon^{htl}_L(-\omega, k),
\]

\[
\text{Im}\epsilon^{htl}_L(\omega, k) = -\text{Im}\epsilon^{htl}_L(-\omega, k).
\]
• The antisymmetric property of $\text{Im} \epsilon_L(\omega = kvx, k)$ about results in the anisotropy of the wake potential
• $\nu \uparrow \rightarrow \text{Im} \epsilon_L(\omega = kvx, k) \uparrow \rightarrow$ the absolute value of $\Phi_2 \rightarrow$ anisotropy enhancement of the wake potential
• The wake structures, ie. the oscillatory or the Lennard-Jones potential are also determined by $\text{Im} \epsilon_L(\omega = kvx, k)$

  $\text{Im} \epsilon_L(\omega = kvx, k)$ influences $\Phi_1$ through $\Delta$. When it is small, its contribution to $\Phi_1$ inappreciable $\rightarrow$ the Lennard-Jones potential. While it is large enough, it changes remarkably $\rightarrow$ oscillatory potential

• The resummation calculation give a correction to the imaginary part of the dielectric function $\rightarrow$ result in the enhancement of the anisotropy of the screening potential.

Conclusion

• The screening potential of a moving test charge → anisotropy in forward-backward direction

• The anisotropic behavior, wake structure, ie. the oscillatory potential or the Lennard-Jones potential are attributed to the imaginary part of the dielectric function.

• The HTL resummation calculation → corrective contribution to imaginary part of the dielectric function → enhance anisotropy