Possible $J^{PC} = 0^{--}$ Charmonium-like State

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OUTLINE

- Part I Introduction
- Part II Currents of $J^{PC} = 0^{--}$ and 0^{-+}
- Part III The Spectral density
- Part IV Numerical Analysis
- Part V Discussion
- Part VI Summary

1 INTRODUCTION

Charmonium Spectroscopy:

Experiments: Belle, BABAR, CDF, D0...

In 2003, Belle reported $X(3872)$ in $B^+ \to K^+ J/\psi \pi^+ \pi^-$ channel.

New States: $X(3872)$, $Y(3930)$, $Z(3930)$, $X(3940)$, $Y(4008)$, $Z_1^+(4050)$, $Y(4140)$, $X(4160)$, $Z_2^+(4250)$, $Y(4260)$, $Y(4360)$, $Z^+(4430)$, $Y(4660)$...

Many interpretations were proposed such as hybrid, molecular or tetraquark state, baryonium state...

Motivation :

- J^{PC} of the quark-antiquark system :

$$P = (-1)^{L+1}, C = (-1)^{L+S},$$

$$J = 0 \Rightarrow L = S, C = (-1)^{L+S} = +1$$

0^{--} is exotic in CQM!

0^{--} tetraquark state :

$qq\bar{q}\bar{q}$: Jiao, Chen, Chen and Zhu, PRD 79, 114034 (2009).

$qc\bar{q}\bar{c}$ state!

2 CURRENTS OF $J^{PC} = 0^{--}$ AND 0^{-+}

Using the diquark-antidiquark construction $(qc)(\bar{q}\bar{c})$, the pseudoscalar tetraquark ($J^P = 0^-$) currents can be constructed :

$$
\begin{aligned}
S_{abcd} &= (q_a^T C c_b)(\bar{q}_c \gamma_5 C \bar{c}_d^T) \, , \\
P_{abcd} &= (q_a^T C \gamma_5 c_b)(\bar{q}_c C \bar{c}_d^T) \, , \\
T_{abcd} &= (q_a^T C \sigma_{\mu\nu} c_b)(\bar{q}_c \sigma^{\mu\nu} \gamma_5 C \bar{c}_d^T) \, , \\
V_{abcd} &= (q_a^T C \gamma_\mu c_b)(\bar{q}_c \gamma^\mu \gamma_5 C \bar{c}_d^T) \, , \\
A_{abcd} &= (q_a^T C \gamma_\mu \gamma_5 c_b)(\bar{q}_c \gamma^\mu C \bar{c}_d^T) .
\end{aligned}
$$

To compose a color singlet current, the diquark and antidiquark should have the same color symmetries:

$$\mathbf{6} \otimes \bar{\mathbf{6}} \text{ or } \bar{\mathbf{3}} \otimes \mathbf{3}$$

$$S_6 = q_a^T C c_b(\bar{q}_a \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_5 C \bar{c}_a^T)\,,$$
$$P_6 = q_a^T C \gamma_5 c_b(\bar{q}_a C \bar{c}_b^T + \bar{q}_b C \bar{c}_a^T)\,,$$
$$T_3 = q_a^T C \sigma_{\mu\nu} c_b(\bar{q}_a \sigma^{\mu\nu} \gamma_5 C \bar{c}_b^T - \bar{q}_b \sigma^{\mu\nu} \gamma_5 C \bar{c}_a^T)\,,$$
$$S_3 = q_a^T C c_b(\bar{q}_a \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_5 C \bar{c}_a^T)\,,$$
$$P_3 = q_a^T C \gamma_5 c_b(\bar{q}_a C \bar{c}_b^T - \bar{q}_b C \bar{c}_a^T)\,,$$
$$T_6 = q_a^T C \sigma_{\mu\nu} c_b(\bar{q}_a \sigma^{\mu\nu} \gamma_5 C \bar{c}_b^T + \bar{q}_b \sigma^{\mu\nu} \gamma_5 C \bar{c}_a^T)\,,$$
$$V_6 = q_a^T C \gamma_\mu c_b(\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T)\,,$$
$$A_3 = q_a^T C \gamma_\mu \gamma_5 c_b(\bar{q}_a \gamma^\mu C \bar{c}_b^T - \bar{q}_b \gamma^\mu C \bar{c}_a^T)\,,$$
$$V_3 = q_a^T C \gamma_\mu c_b(\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T)\,,$$
$$A_6 = q_a^T C \gamma_\mu \gamma_5 c_b(\bar{q}_a \gamma^\mu C \bar{c}_b^T + \bar{q}_b \gamma^\mu C \bar{c}_a^T)\,.$$

Taking the charge-conjugation transformation :

$$\mathbb{C}S_6\mathbb{C}^{-1} = P_6\,, \mathbb{C}S_3\mathbb{C}^{-1} = P_3\,, \mathbb{C}T_3\mathbb{C}^{-1} = T_3\,,$$
$$\mathbb{C}V_3\mathbb{C}^{-1} = A_3\,, \mathbb{C}V_6\mathbb{C}^{-1} = A_6\,, \mathbb{C}T_6\mathbb{C}^{-1} = T_6\,.$$

We get four currents with $J^{PC} = 0^{--}$:

$$\begin{aligned}
\eta_1 = S_6 - P_6 = & \; q_a^T C c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_5 C \bar{c}_a^T) \\
& - q_a^T C \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T + \bar{q}_b C \bar{c}_a^T)\,, \\
\eta_2 = V_6 - A_6 = & \; q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T) \\
& - q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma^\mu C \bar{c}_b^T + \bar{q}_b \gamma^\mu C \bar{c}_a^T)\,, \\
\eta_3 = V_3 - A_3 = & \; q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T) \\
& - q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma^\mu C \bar{c}_b^T - \bar{q}_b \gamma^\mu C \bar{c}_a^T)\,, \\
\eta_4 = S_3 - P_3 = & \; q_a^T C c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_5 C \bar{c}_a^T) \\
& - q_a^T C \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T - \bar{q}_b C \bar{c}_a^T)\,.
\end{aligned}$$

And six currents with $J^{PC} = 0^{-+}$:

$$
\begin{aligned}
\eta_5 = S_6 + P_6 &= q_a^T C c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma_5 C \bar{c}_a^T) \\
&\quad + q_a^T C \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T + \bar{q}_b C \bar{c}_a^T) \,, \\
\eta_6 = \quad T_3 \quad &= q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \sigma^{\mu\nu} \gamma_5 C \bar{c}_b^T - \bar{q}_b \sigma^{\mu\nu} \gamma_5 C \bar{c}_a^T) \\
\eta_7 = V_6 + A_6 &= q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T + \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T) \\
&\quad + q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma^\mu C \bar{c}_b^T + \bar{q}_b \gamma^\mu C \bar{c}_a^T) \,, \\
\eta_8 = V_3 + A_3 &= q_a^T C \gamma_\mu c_b (\bar{q}_a \gamma^\mu \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma^\mu \gamma_5 C \bar{c}_a^T) \\
&\quad + q_a^T C \gamma_\mu \gamma_5 c_b (\bar{q}_a \gamma^\mu C \bar{c}_b^T - \bar{q}_b \gamma^\mu C \bar{c}_a^T) \,, \\
\eta_9 = S_3 + P_3 &= q_a^T C c_b (\bar{q}_a \gamma_5 C \bar{c}_b^T - \bar{q}_b \gamma_5 C \bar{c}_a^T) \\
&\quad + q_a^T C \gamma_5 c_b (\bar{q}_a C \bar{c}_b^T - \bar{q}_b C \bar{c}_a^T) \,, \\
\eta_{10} = \quad T_6 \quad &= q_a^T C \sigma_{\mu\nu} c_b (\bar{q}_a \sigma^{\mu\nu} \gamma_5 C \bar{c}_b^T + \bar{q}_b \sigma^{\mu\nu} \gamma_5 C \bar{c}_a^T)
\end{aligned}
$$

3 THE SPECTRAL DENSITY

Up to dimension 8, the spectral density $\rho_i(s)$ at the quark-gluon level reads:

$$\rho^{OPE} = \rho^{pert}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{mix}(s) + \rho^{\langle\bar{q}q\rangle^2}$$

For these expressions, the integration limits are:

$$\alpha_{max} = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2}, \quad \alpha_{min} = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2}$$

$$\beta_{max} = 1 - \alpha, \quad \beta_{min} = \frac{\alpha m_c^2}{\alpha s - m_c^2}.$$

$$\rho_2^{pert}(s) = \frac{1}{2^5\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{\beta_{max}}\frac{(1-\alpha-\beta)^2}{\beta^3}[(\alpha+\beta)m_c^2-3\alpha\beta s][(\alpha+\beta)m_c^2-\alpha\beta s]^3\,,$$

$$\rho_2^{\langle\bar{q}q\rangle}(s) = 0\,,$$

$$\rho_2^{\langle G^2\rangle}(s) = \frac{\langle g^2G^2\rangle}{2^5\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{\beta_{max}}\{\frac{(1-\alpha-\beta)^2m_c^2}{3\alpha}[2(\alpha+\beta)m_c^2-3\alpha\beta s] + \frac{5(1-\alpha-\beta)}{4\beta}[(\alpha+\beta)m_c^2-2\alpha\beta s][(\alpha+\beta)m_c^2-\alpha\beta s]\}\,,$$

$$\rho_2^{mix}(s) = 0\,,$$

$$\rho_2^{\langle\bar{q}q\rangle^2}(s) = -\frac{4m_c^2\langle\bar{q}q\rangle^2}{3\pi^2}\sqrt{1-4m_c^2/s}\,, \quad (1)$$

$$\Pi_2^{mix\langle\bar{q}q\rangle}(M_B^2) = -\frac{m_c^2\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle}{6\pi^2}\int_0^1\frac{d\alpha}{\alpha}(\frac{4m_c^2}{\alpha M_B^2}-5)e^{-\frac{m_c^2}{\alpha(1-\alpha)M_B^2}}\,,$$

$$\Pi_2^{\langle G^3\rangle}(M_B^2) = -\frac{\langle g^3fG^3\rangle}{3\times 2^6\pi^6}\int_0^1 d\alpha\int_0^{\beta_{max}}d\beta\{\frac{\ln(\alpha\beta(1-\alpha-\beta)M_B^4)}{\alpha\beta}[12(\alpha\beta M_B^2)^2 + 6\alpha\beta M_B^2(\alpha+\beta)m_c^2+(\alpha+\beta)^2m_c^4] + \frac{(1-\alpha-\beta)^2m_c^2}{\alpha^4}[2\alpha\beta M_B^2+(\alpha+\beta)m_c^2] - \frac{(1-\alpha-\beta)^2}{2\alpha^3}[3\alpha\beta M_B^4+M_B^2(\alpha+\beta)m_c^2]\}e^{-\frac{(\alpha+\beta)m_c^2}{\alpha\beta M_B^2}}\,.$$

$$\rho_6^{pert}(s) = \frac{3}{2^6\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{\beta_{max}}\frac{(1-\alpha-\beta)^2}{\beta^3}[(\alpha+\beta)m_c^2-3\alpha\beta s][(\alpha+\beta)m_c^2-\alpha\beta s]^3\,,$$

$$\rho_6^{\langle\bar{q}q\rangle}(s) = 0\,,$$

$$\rho_6^{\langle G^2\rangle}(s) = \frac{\langle g^2G^2\rangle}{2^6\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{\beta_{max}}\{\frac{(1-\alpha-\beta)^2m_c^2}{\alpha}[2(\alpha+\beta)m_c^2-3\alpha\beta s]$$
$$+\frac{(1-\alpha-\beta)^2+2\alpha\beta}{4\beta^2}[(\alpha+\beta)m_c^2-2\alpha\beta s][(\alpha+\beta)m_c^2-\alpha\beta s]\,,$$

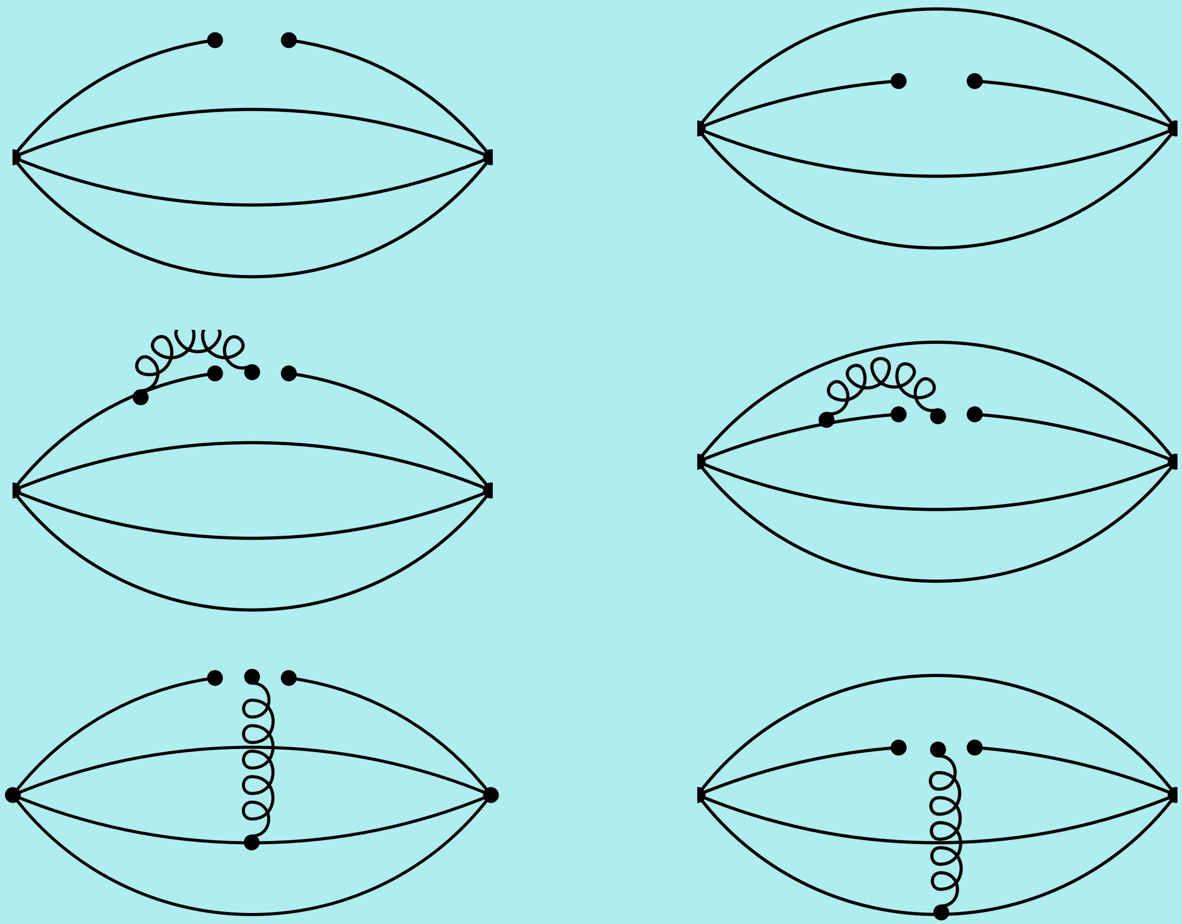
$$\rho_6^{mix}(s) = 0\,,$$

$$\rho_6^{\langle\bar{q}q\rangle^2}(s) = -\frac{2m_c^2\langle\bar{q}q\rangle^2}{\pi^2}\sqrt{1-4m_c^2/s}\,, \qquad (2)$$

$$\Pi_6^{mix\langle\bar{q}q\rangle}(M_B^2) = -\frac{m_c^2\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle}{3\pi^2}\int_0^1\frac{d\alpha}{\alpha}(\frac{3m_c^2}{\alpha M_B^2}-1)e^{-\frac{m_c^2}{\alpha(1-\alpha)M_B^2}}\,,$$

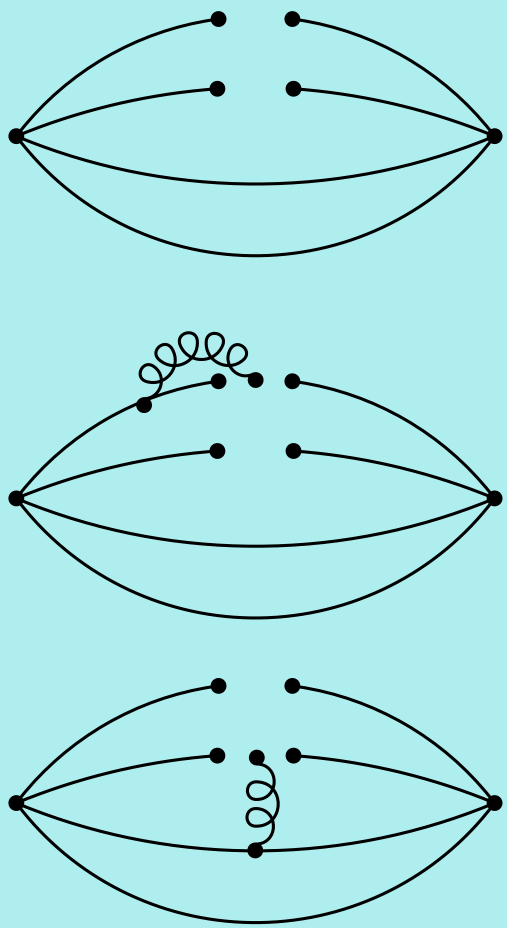
$$\Pi_6^{\langle G^3\rangle}(M_B^2) = -\frac{\langle g^3fG^3\rangle}{3\times 2^7\pi^6}\int_0^1 d\alpha\int_0^{\beta_{max}}d\beta\{\frac{\ln(\alpha\beta(1-\alpha-\beta)M_B^4)}{\alpha\beta}[12(\alpha\beta M_B^2)^2+$$
$$6\alpha\beta M_B^2(\alpha+\beta)m_c^2+(\alpha+\beta)^2m_c^4]+\frac{3(1-\alpha-\beta)^2m_c^2}{\alpha^4}$$
$$[2\alpha\beta M_B^2+(\alpha+\beta)m_c^2]-\frac{3(1-\alpha-\beta)^2}{2\alpha^3}[3\alpha\beta M_B^4+M_B^2(\alpha+\beta)m_c^2]$$
$$+\frac{2}{1-\alpha-\beta}[3\alpha\beta M_B^4+M_B^2(\alpha+\beta)m_c^2]\}e^{-\frac{(\alpha+\beta)m_c^2}{\alpha\beta M_B^2}}\,.$$

- Both condensates $\langle\bar{q}q\rangle$ and $\langle\bar{q}g_s\sigma\cdot Gq\rangle$ vanish for the special Lorentz structures of the currents:



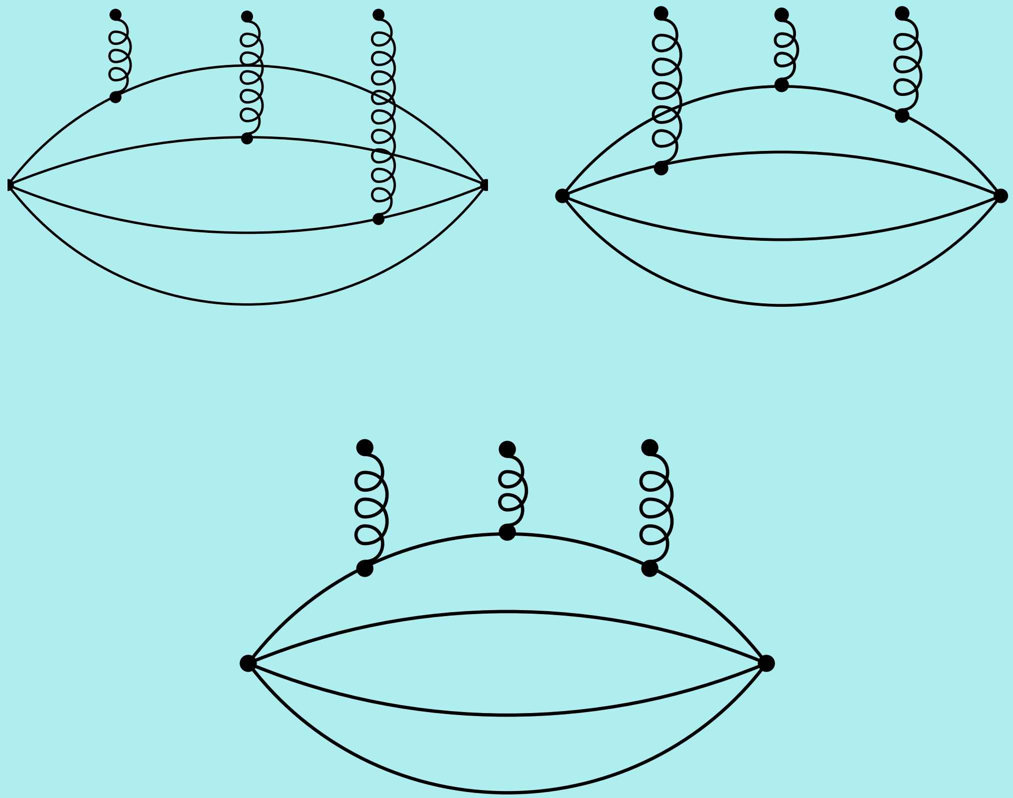
Vanish!

- The four quark condensates $\langle\bar{q}q\rangle^2$ and $\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle$ are proportional to the charm quark mass and can not be omitted :



Can not be Omitted!

- The tri-gluon condensates $\langle g_s^3 f^{abc} G^a G^b G^c \rangle$: The first and second classes vanishe because of the special color and Lorentz structures of the currents. The third class is much smaller than other condensates:



4 NUMERICAL ANALYSIS

We use the following values of the quark masses and various condensates in the QCD sum rule analysis:

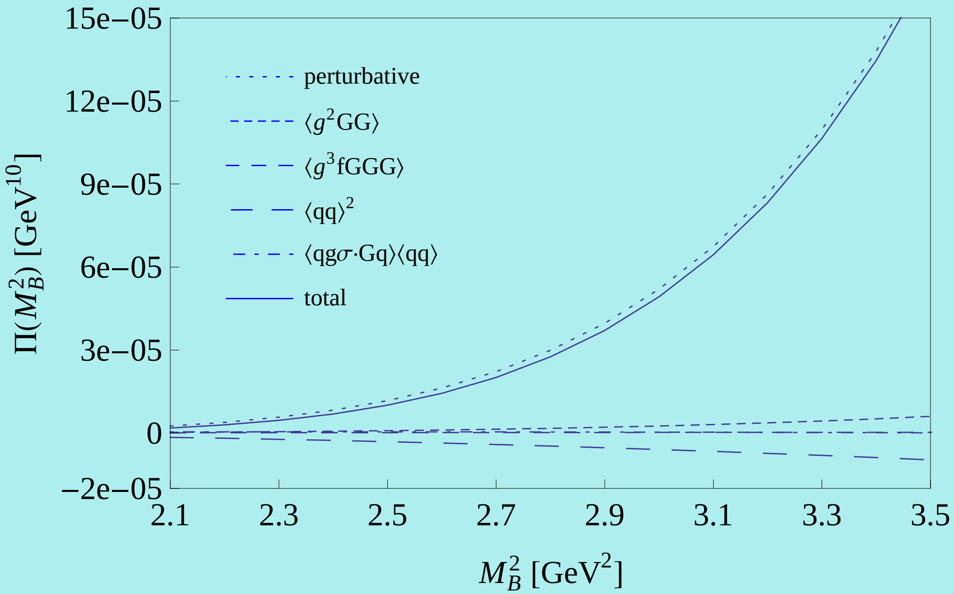
$$
\begin{aligned}
m_c(m_c) &= (1.23 \pm 0.09)\ \text{GeV}\,, \\
m_b(m_b) &= (4.20 \pm 0.07)\ \text{GeV}\,, \\
\langle \bar{q}q \rangle &= -(0.23 \pm 0.03)^3\ \text{GeV}^3\,, \\
\langle \bar{q}g_s\sigma \cdot Gq \rangle &= -M_0^2 \langle \bar{q}q \rangle\,, \\
M_0^2 &= (0.8 \pm 0.2)\ \text{GeV}^2\,, \\
\langle \bar{s}s \rangle / \langle \bar{q}q \rangle &= 0.8 \pm 0.2\,, \\
\langle g_s^2 GG \rangle &= 0.88\ \text{GeV}^4\,, \\
\langle g_s^3 fGGG \rangle &= 0.045\ \text{GeV}^6\,.
\end{aligned}
$$

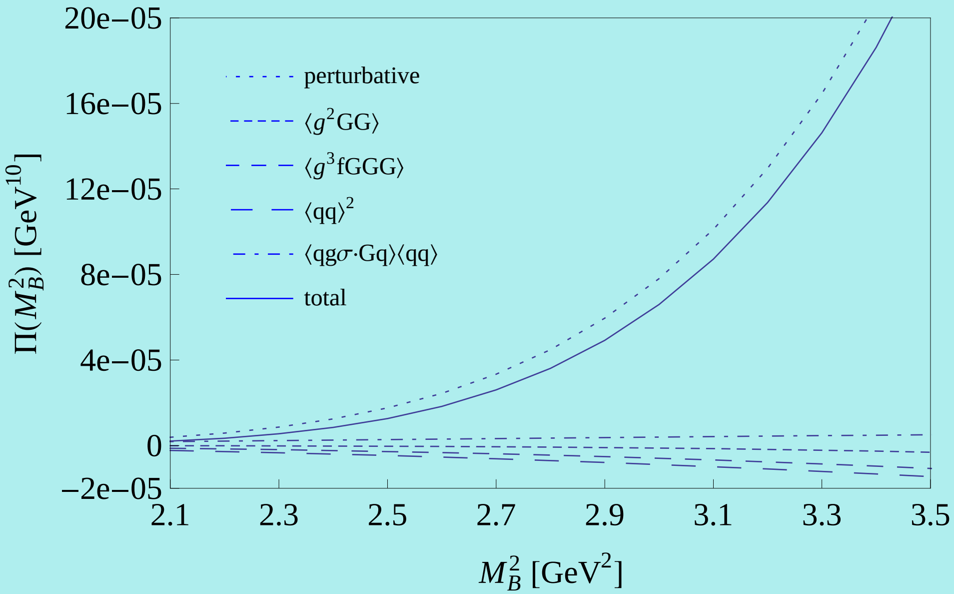
The pole contribution(PC):

$$\frac{\int_{4m_c^2}^{s_0} ds e^{-s/M_B^2}\rho(s)}{\int_{4m_c^2}^{\infty} ds e^{-s/M_B^2}\rho(s)}.$$

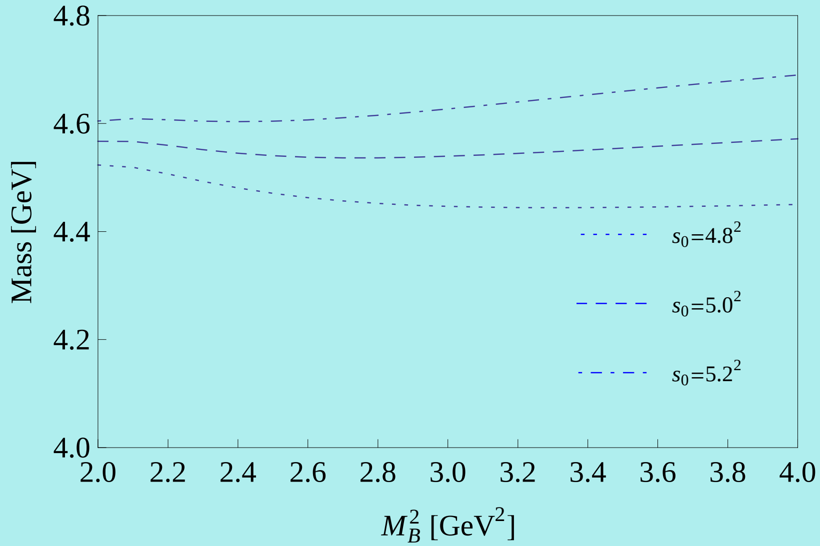
- Requiring the pole contribution is larger than 40%, we get the upper bound $M^2_{\rm max}$ of the Borel parameter M_B^2;
- The convergence of the OPE leads to the lower bound $M^2_{\rm min}$ of the Borel parameter M_B^2.

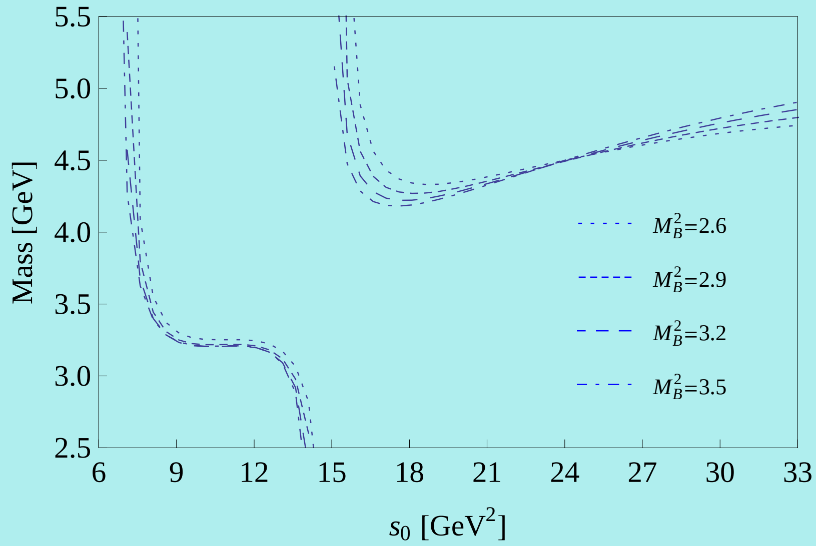
OPE Convergence :

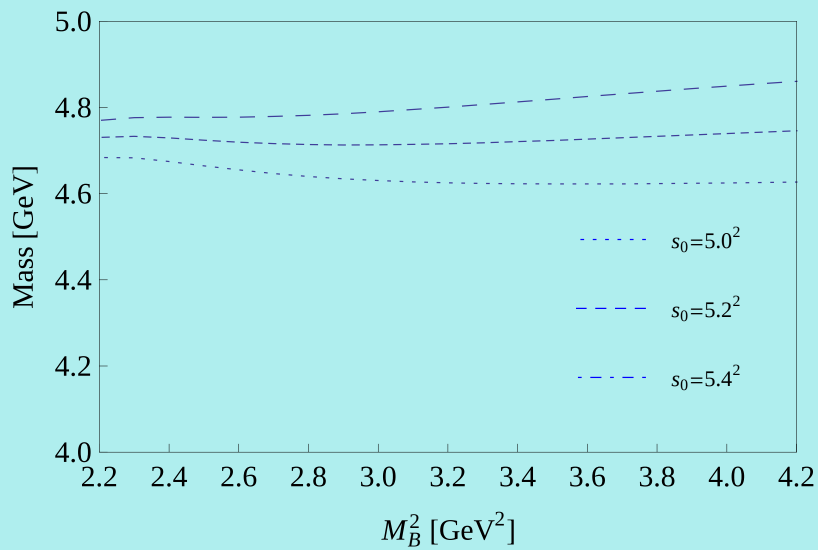


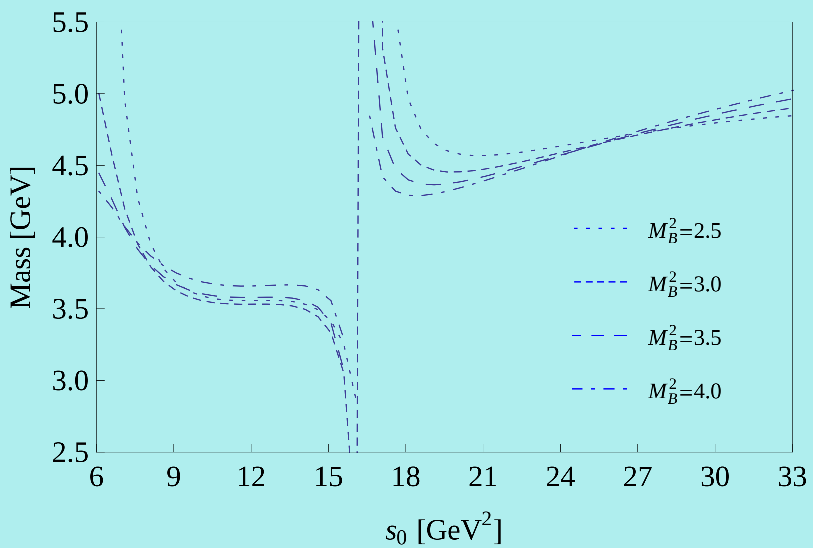


Mass figures:









$qc\bar{q}\bar{c}$ system :

	Currents	$s_0(\mathrm{GeV}^2)$	$[M^2_{\min}, M^2_{\max}]$	$m_X(\mathrm{GeV})$	PC(%)
$J^{PC}=0^{--}$	η_1	25	$2.4 \sim 3.6$	$-$	-
	η_2	25	$2.4 \sim 3.7$	4.55 ± 0.11	46.3
	η_3	25	$2.4 \sim 3.7$	$-$	-
	η_4	25	$2.4 \sim 3.7$	4.55 ± 0.11	45.9
$J^{PC}=0^{-+}$	η_5	25	$2.4 \sim 3.6$	$-$	-
	η_6	27	$2.4 \sim 4.1$	4.72 ± 0.10	53.8
	η_7	25	$2.4 \sim 3.8$	$-$	-
	η_8	25	$2.4 \sim 3.7$	$-$	-
	η_9	25	$2.4 \sim 3.7$	4.55 ± 0.11	45.9
	η_{10}	27	$2.4 \sim 4.2$	4.67 ± 0.10	56.8

$\sqrt{s_0} = 5.0$ GeV and $M_B^2 = 3.5\mathrm{GeV}^2$

$qb\bar{q}\bar{b}$ system :

	Currents	$s_0(\text{GeV}^2)$	$[M^2_{\text{min}}, M^2_{\text{max}}]$	$m_{X_b}(\text{GeV})$	PC(%)
$J^{PC}=0^{--}$	η_{1b}	11.2^2	$6.4 \sim 9.4$	$-$	-
	η_{2b}	11.2^2	$6.4 \sim 9.5$	10.64 ± 0.12	45.2
	η_{3b}	11.2^2	$6.4 \sim 9.5$	$-$	-
	η_{4b}	11.2^2	$6.4 \sim 9.5$	10.64 ± 0.12	45.1
$J^{PC}=0^{-+}$	η_{5b}	11.2^2	$6.4 \sim 9.4$	$-$	-
	η_{6b}	11.2^2	$6.4 \sim 9.4$	10.67 ± 0.11	44.2
	η_{7b}	11.2^2	$6.4 \sim 9.7$	$-$	-
	η_{8b}	11.2^2	$6.4 \sim 9.6$	$-$	-
	η_{9b}	11.2^2	$6.4 \sim 9.5$	10.64 ± 0.12	45.1
	η_{10b}	11.2^2	$6.4 \sim 9.5$	10.64 ± 0.11	45.6

$\sqrt{s_0} = 11.2$ GeV and $M_B^2 = 9.0\text{GeV}^2$

5 Discussion

0^{--} **charmonium-like state:**

- Our result: $m = 4.5$GeV;
- K. T. Chao, Nucl. Phys. B 169, 281(1980);
  $m = 4.4$GeV.
  $m = 4.1$GeV.
- Ebert *et al.* Eur. Phys. J. C 58, 399(2008);
  $m = 4.3$GeV.
  ...

- The possible decay modes :

I^G	S-wave	P-wave
0^-	$D^*(2007)^0\bar{D}_1(2420)^0 + c.c.$, $D_0^*(2400)^0\bar{D}^0(1865) + c.c.$, $\omega(782)\chi_{c1}(1P)$, $J/\psi f_1(1285)$	$D^0(1865)\bar{D}^*(2007)^0 + c.c.$, $D^*(2007)^0\bar{D}^*(2007)^0$, $J/\psi\eta$, $J/\psi\eta'$, $\psi(2S)\eta$, $\eta_c(1S)\omega$, $\eta_c(2S)\omega$, $h_c(1P)\sigma$, $h_c(1P)f_0(980)$
1^+	$D^*(2007)^0\bar{D}_1(2420)^0 + c.c.$, $D_0^*(2400)^0\bar{D}^0(1865) + c.c.$, $\rho(770)\chi_{c1}(1P)$, $J/\psi a_1(1260)$	$D^0(1865)\bar{D}^*(2007)^0 + c.c.$, $D^*(2007)^0\bar{D}^*(2007)^0$, $\eta_c(1S)\rho$, $\eta_c(2S)\rho$, $h_c(1P)a_0(980)$ $J/\psi\pi$, $J/\psi\pi_1(1400)$, $\psi(2S)\pi$

- Experimental Search : Super-B factories, PANDE, LHC, RHIC ?

6 SUMMARY

- We construct the charmonium-like tetraquark interpolating currents with $J^{PC} = 0^{--}$ and 0^{-+} using diquark and antidiquark fields.
- The four quark condensate $\langle\bar{q}q\rangle^2$ becomes the dominant power correction. Both condensates $\langle\bar{q}q\rangle$ and $\langle\bar{q}\sigma\cdot Gq\rangle$ vanish.
- Within the framework of the SVZ sum rule, the extracted masses are around 4.5 GeV for the 0^{--} charmonium-like state and 4.6 GeV for the 0^{-+} charmonium-like state while their bottomonium-like analogues lie around 10.6 GeV.
- We also discuss the possible decay and the experimental search of the 0^{--} charmonium-like state.

THANK YOU!