Landau Fermi-liquid parameters of the strongly interacting fermions system

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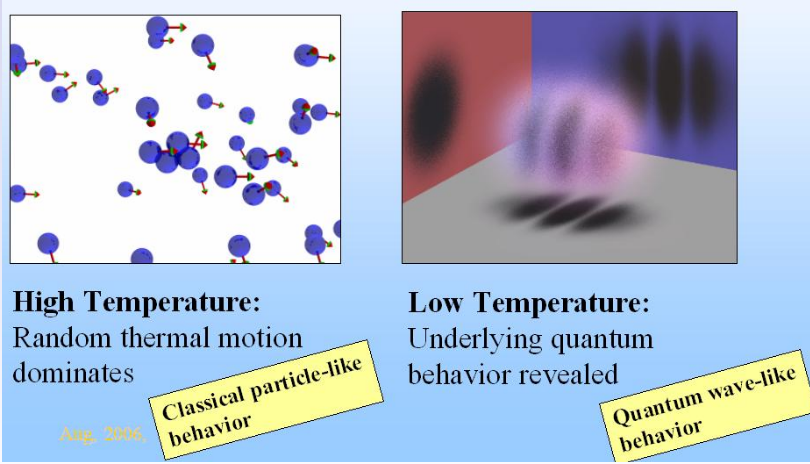
Contents

1, Motivations: why study unitary Fermi gases?

Answer: coherent quantum phenomena



High Temperature:
Random thermal motion dominates

Classical particle-like behavior

Aug. 2006,

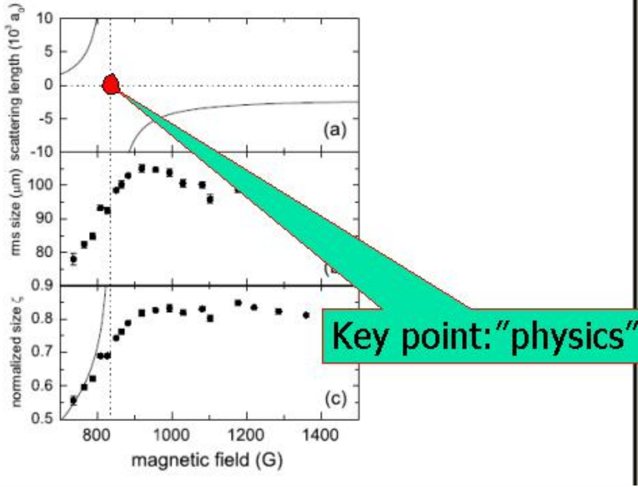
Low Temperature:
Underlying quantum behavior revealed

Quantum wave-like behavior

Feshbach resonance

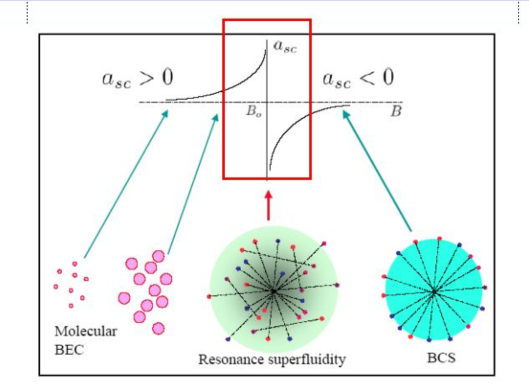
Bare scattering amplitude $f_0^s \sim \frac{i}{k}$ at "unitarity".

Contact interaction: infrared **essential** singularity.



Key point:"physics"

Fermions matter with large scattering length



An intriguing problem:

Strongly coupling or correlating limit physics

Ultra-cold atomic fermions

- Platform
  To explore strongly interacting/non-perturbative many-body physics with dilute ultra-cold fermions atoms
- interdisciplinary
  Attract much attention in recent years

2, Universal unitary Fermi gas thermodynamics

Dynamics & Quantum effects

- Finite interaction: thermodynamic quantities as a function of a, T, n.
- Unitary limit: zero-energy bound, $|a| = \infty$
  The dynamical details are erased in the thermodynamical quantities.

Unitary fermions physics

1. "Conformal invariance" Similar to ideal gas law? For example,
    - $T = 0$
      $P = \frac{2}{5}n\mu = \xi\frac{3}{5}n\varepsilon_f$
      Exact value ξ? "Bertsch Problem" of neutron nuclear matter, 1999
    - $P = \frac{2}{3}\frac{E}{V}$

Unitary fermions physics

1. "Conformal invariance" Similar to ideal gas law? For example,
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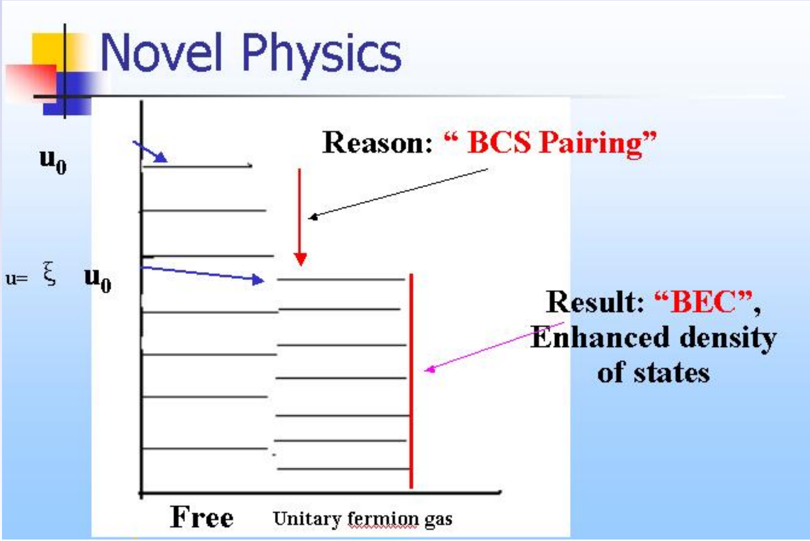
    - $P = \frac{2}{3}\frac{E}{V}$
2. Low viscosity $\eta/s \geq \frac{1}{4\pi}$? (Ads/CFT conjuncture) Perfect quantum liquid

Non-perturbative many-body topic

Novel rearrangement effects



Novel Physics

Reason: “ BCS Pairing”

Result: “BEC”, Enhanced density of states

“Quantum levels crossing” effects

3, Landau Fermi-liquid parameters

- Original formalism:
  L.D. Landau, Sov. Phys.-JETP **3**, 920 (1957).
  L.D. Landau, Sov. Phys.-JETP **5**, 101 (1957).
- Review books:
  D. Pines; G. Baym and C.J. Pethick
- **Ground stone**
  1. Neutron nuclear matter, condensed matter physics
  2. Related with the neutrinosphere physics
  3. Model independent! Experimentally validate.

Successful:
Description of low temperature ^{3}He fluid;
Heavy fermions physics in condensed matter physics;
etc.

Central idea: quasi-particle concept

1. There is interaction between quasi-particles

$$E/V \neq \int \epsilon(\mathbf{p}) n(\mathbf{p}) d\mathbf{p}.$$

2. The variance of E/V according to δn up to second order

$$\delta E/V = \int \epsilon^0(\mathbf{p}) \delta n(\mathbf{p}) d\mathbf{p} + \frac{1}{2} \int\int f(\mathbf{p}, \mathbf{p}') \delta n(\mathbf{p}) \delta n(\mathbf{p}') d\mathbf{p} d\mathbf{p}'.$$

3. Entropy density

$$S/V = -k_B \int \left(n \ln n + (1-n) \ln(1-n) \right) d\mathbf{p}.$$

mean-field theory formalism

Landau parameters

Galilean invariance

1, m^*/m is related with the interaction function **f**:
Heat capacity (response function):

$$m^*/m = 1 + \frac{1}{3}F_1^s. \tag{1}$$

$$C_V/V \propto \frac{m^*}{m}T.$$

related to entropy (temperature)
2, Compressibility(response function): sound velocity ∼ EOS

$$c^2 = \frac{1+F_0^s}{1+F_1^s/3}c_i^2. \tag{2}$$

$$c_i^2 = \frac{1}{3}v_f^2. \quad v_f = \frac{p_f}{m}.$$

The methods for fixing F_0^s, F_1^s

- Experimentally determined:
  ^3H_e: $m^* \sim 3.1m$.
- "**Calculated**" for weak coupling:
  Interaction vertex's correction etc. with various coupled Galitskii, Bethe-Sepeter or Bethe-Goldstone integral equations
  (Abrikosov, Gorkov and Dzyaloshinski, 1963; Fetter and Walecka,1974)

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Lowest order m^*

$$\frac{m^*}{m} = 1 + \frac{8(7\ln 2 - 1)}{15\, 3^{1/3}\, \pi^{5/3}} (k_f a)^2 + ?.$$

"dilute parameter" $\sim k_f a$

weak: $|k_f a| \ll 1$; strong : $|k_f a| \geq 1$

Why?

Why?

Crucial for phase diagram, transport/response properties

The existed results for m^*/m ($\sim F_1^s$) of unitary Fermi system

$\frac{m^*}{m}$ =?

1. *Päde approximation*: $m^*/m \sim 2.5$ at unitarity,PRC **60**, 054311 (1999).
2. *Numerical fitting* $m^*/m \sim 1.04 - 1.17$
   PRL **97**, 200403 (2006).
   PRL **98**, 180402 (2007).
   PRL **100**, 030401 (2008).
   etc...
   "Uncertain" updating $\sim 1.04 - 1.18$.

New $m^*/m \sim 1.2(2)$, Nature 463, 1057 (2010)

Toy model

$$H = -\int d^3x\psi^*_\alpha(x)(\frac{\nabla^2}{2m})\psi_\alpha(x) + \frac{U}{2}\int d^3x\psi^*_\alpha(x)\psi^*_\beta(x)\psi_\beta(x)\psi_\alpha(x).$$

Scattering length with low energy Born approximation(3-D):

$$U \equiv \frac{4\pi a}{m}.$$

Four fermions contact interaction ⇒ Bethe-Peierls, Hubbard, ...,

Ising-like: Terrible
$1 - D$, $2 - D$, Bethe ansatz (valid only for weak coupling?)
3-D?

How? Generalized Dyson-Scwhinger equations

Infinite Feynman diagramms

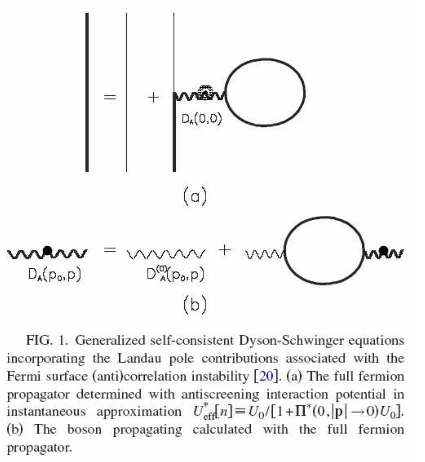

FIG. 1. Generalized self-consistent Dyson-Schwinger equations incorporating the Landau pole contributions associated with the Fermi surface (anti)correlation instability [20]. (a) The full fermion propagator determined with antiscreening interaction potential in instantaneous approximation $U^*_{\rm eff}[n]\equiv U_0/[1+\Pi^*(0,|\mathbf{p}|\to 0)U_0]$. (b) The boson propagating calculated with the full fermion propagator.

PRA **76**, 033617 (2007).

non-relativistic version of CPL, CTP(2007)

Formalism

Non-linear transformation

J. Stat. Mech.08 (2009) Lett. 08002

Medium **"scaling" effective action**:

$$\tilde{H} = -\int d^3x \tilde{\psi}_\alpha^*(x)(\frac{\nabla^2}{2m} - \mu_r[n,T])\tilde{\psi}_\alpha(x) + \frac{\tilde{U}_{\rm eff}[n,T]}{2}\int d^3x \tilde{\psi}_\alpha^*(x)\tilde{\psi}_\beta^*(x)\tilde{\psi}_\beta(x)\tilde{\psi}_\alpha(x).$$

Formalism

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Particle-hole symmetry is explicitly broken;

Anti-**correlation potential**

$$\tilde{U}_{eff}[n,T] = \frac{U_0}{1-\frac{1}{2}m_D^2 U_0},$$

$$m_D^2 = 2\chi' = \left(\frac{\partial n}{\partial \mu^*}\right)_T = -\Pi(0, |\mathbf{k}| \to 0).$$

(Crucial: fluctuation and correlation physics)

Realistic $\Omega(T,\mu) = -PV$

made of **parametric equations**

$$P = P_{\text{ideal}}(T,\mu^*) + \frac{\pi a_{\text{eff}}}{m} n^2 + \mathcal{C}\left(\frac{2\pi a_{\text{eff}}}{m}\right)^2 n^3, \qquad (3)$$

$$\mu = \mu^* + \frac{2\pi a_{\text{eff}}}{m} n + \mathcal{C}\left(\frac{2\pi a_{\text{eff}}}{m}\right)^2 n^2, \qquad (4)$$

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$$\mu = \mu^* + \frac{2\pi a_{\text{eff}}}{m} n + \mathcal{C}\left(\frac{2\pi a_{\text{eff}}}{m}\right)^2 n^2, \qquad (4)$$

or $f = F/V = \epsilon - Ts$

$$s = S/V = s_{\text{ideal}} + \mathcal{D}\left(\frac{2\pi a_{\text{eff}}}{m}\right)^2 n^2, \qquad (5)$$

$$\epsilon = E/V = 2\int_k \frac{k^2}{2m} f_k + \frac{\pi a_{\text{eff}}}{m} n^2 + T\mathcal{D}\left(\frac{2\pi a_{\text{eff}}}{m}\right)^2 n^2, \qquad (6)$$

Non-Gaussian physics through **implicit variable** μ^*

1. **Implicit variable**: effective chemical potential μ^* or effective fugacity $z' = e^{\mu^*/T}$
2. In-medium "**correlation factors**" $\mathcal{C}$ and $\mathcal{D}$

$$\mathcal{C} = \frac{1}{2}\left(\frac{\partial \chi'}{\partial n}\right)_T .$$

$$\mathcal{D} = -\frac{1}{2}\left(\frac{\partial \chi'}{\partial T}\right)_n .$$

$$= -\frac{1}{4T}\chi' + \frac{3\mathcal{C}}{2T}n$$

3. low-T expansion with "Sommerfeld Lemma" ⇒ F_0^s, F_1^s

In terms of T/μ^* instead of T/μ

Final results for F_1^s and F_0^s

Compared with Landau Fermi-liquid theory, we can have

1. General

$$\begin{aligned}\frac{m^*}{m}[k_f a] &= 1+\frac{1}{3}F_1^s \\ &= 1+\frac{4(k_f a)^2}{9(\pi-2ak_f)^2}. \qquad (7)\\ \frac{1+F_0^s}{1+\frac{1}{3}F_1^s}[k_f a] &= 1+\frac{2ak_f}{\pi-2ak_f}+\frac{20(k_f a)^2}{9(\pi-2ak_f)^2} \\ &\quad +\frac{8(k_f a)^3}{9(\pi-2ak_f)^3} \\ &= \xi[k_f a] \qquad (8)\end{aligned}$$

2. At uniarity, F_0^s and F_1^s universal constants:

$$\xi=\frac{4}{9}; \qquad \frac{m^*}{m}=\frac{10}{9}$$

Main results:

consistent with existed theoretical/experimental attempts

- Universal F_0^s and F_1^s fixed analytically
- $\Rightarrow \frac{m^*}{m} = \frac{10}{9}$ "real part"; $\xi = \frac{4}{9}$("imaginary part")

Correlation function or selfenergy?

4, Conclusions

Methodology

4, Conclusions

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1. Quasi-linear approximation: non-Gaussian effects with in-medium non-local correlations

4, Conclusions

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1. Quasi-linear approximation: non-Gaussian effects with in-medium non-local correlations

highly nonlinear-turbulent.

4, Conclusions

Methodology

1. Quasi-linear approximation: non-Gaussian effects with in-medium non-local correlations highly nonlinear-turbulent.
2. Various infinite "loop" diagrams: low order mixed with high order

4, Conclusions

Methodology

1. Quasi-linear approximation: non-Gaussian effects with in-medium non-local correlations
   highly nonlinear-turbulent.
2. Various infinite “loop” diagrams: low order mixed with high order
3. Dynamical **mixed with** statistical fluctuation/correlation: induced high partial wave contribution

Thank you!

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