# Polarized Deep Inelastic Scattering and Deeply Virtual Compton Scattering From Gravity/Gauge Duality

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2010.04.19

Gao & Mou arXiv:1003.3066
Gao & Xiao arXiv:0912.4333 PRD,2010
Gao & Xiao arXiv:0904.2870 PRD,2009

# Outline

# AdS/CFT correspondence

CFT: $\mathcal{N} = 4$ SYM theory in $3 + 1$ dimension $\left(N_c, \lambda = g_{\rm YM}^2 N_c\right)$

$$\Updownarrow$$

AdS: Type II B string theory on $\mathrm{AdS}_5 \times S^5$ $\left(g_s, R = (4\pi g_s N_c)^{1/4} l_s\right)$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) + R^2 d\Omega_5^2$$

Parameters Correspondence: $g_s \sim g_{\rm YM}^2$

In the limit: $\lambda \to \infty \; (g_s < 1, N_c \to \infty)$

$$\Rightarrow \qquad \frac{R}{l_s} \sim (4\pi\lambda)^{1/4} >> 1 \qquad \text{Classic Supergravity}$$

# Holographic Dictionary

| CFT(Gauge side: operator) | AdS(Gravity side: field) |
|---|---|
| $\mathrm{Tr}(F^2)$ | $\phi(x,z)$ (Dilaton) |
| $T^{\mu\nu}$ (Energy momentum tensor) | $h^{\mu\nu}(x,z)$ (Graviton) |
| $J^{\mu}$ (Conserved Current) | $A^{\mu}(x,z)$ (Gauge Field) |

Identify the partition functions of the two different theory:

$$\int \exp\left\{iS_{4D} + \int d^4x\phi_0\mathcal{O}\right\} = \int \exp\left\{iS_{\mathrm{bulk}}[\phi]\right\}\Big|_{\phi(x,0)=\phi_0}$$

Calculate the correlation functions in CFT through the on-shell supergravity action:

$$\langle\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\rangle = \frac{\delta}{\delta\phi_0(x_1)}\cdots\frac{\delta}{\phi_0(x_n)}\exp\left\{i\tilde{S}_{\mathrm{bulk}}[\phi_0]\right\}$$

# AdS/QCD

**Try to obtain 5D dual gravity theory for QCD!**

Top-down approach: From string theory

D3-D7 system (Kruczenski, Mateos, Myers, Winters '04).

D4-D8 system (Sakai, Sugumoto. '04)

. . . . . ..

Bottom-up approach: phenomenological

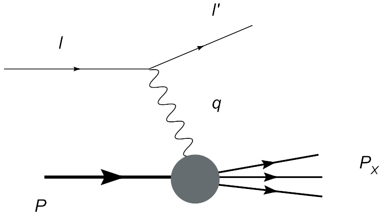
Hard wall model: introduce a sharp IR cut for confinement. (Polchinski & Strassler '00)

Soft wall model: introduce a smooth running dilaton field. (Erich, Katz, Son & Stephanov '05)

# Polarized Deep Inelastic Scattering

Hadronic Tensor :

$$W_{\mu\nu} \equiv \langle P,S|[J_\mu(q),J_\nu(0)]|P,S\rangle$$
$$= W^{(S)}_{\mu\nu}(q,P) + i\, W^{(A)}_{\mu\nu}(q,P,S)$$




Decomposition:

$$\begin{aligned} W^{(S)}_{\mu\nu} &= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[F_1(x,q^2) + \frac{MS\cdot q}{2P\cdot q} g_5(x,q^2)\right] \\ &\quad - \frac{1}{P\cdot q}\hat{P}_\mu \hat{P}_\nu \left[F_2(x,q^2) + \frac{MS\cdot q}{P\cdot q} g_4(x,q^2)\right] - \frac{M}{2P\cdot q}\left(\hat{P}_\mu S_{\perp\nu} + \hat{P}_\nu S_{\perp\mu}\right) g_3(x,q^2), \\ W^{(A)}_{\mu\nu} &= -\frac{M\,\varepsilon_{\mu\nu\rho\sigma}\, q^\rho}{P\cdot q}\left\{S^\sigma\, g_1(x,q^2) + S^\sigma_\perp g_2(x,q^2)\right\} - \frac{\varepsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma}{2P\cdot q} F_3(x,q^2), \end{aligned}$$

Kinetic definitions:

$$x \equiv -\frac{q^2}{2P\cdot q}\,, \quad \hat{P}_\mu \equiv P_\mu - \frac{P\cdot q}{q^2} q_\mu \quad S_{\perp\mu} \equiv S_\mu - \frac{S\cdot q}{P\cdot q} P_\mu\,.$$

# Structure Functions from PQCD

## PQCD In the Bjorken limit $q^2 \to \infty$, with $x$ fixed .

**Unpolarized structure functions :** $F_1(x)=\frac{1}{2x}F_2(x)=\frac{1}{2}\sum_i e_i^2\left[f_i^{\uparrow}(x)+f_i^{\downarrow}(x)\right]$

Momentum sum rule : $\int_0^1 dx\, x\left[f_i^{\uparrow}(x)+f_i^{\downarrow}(x)\right] = \frac{3}{16+3n_i}$

**Polarized structure functions :** $g_1(x)=\frac{1}{2}\sum_i e_i^2\left[f_i^{\uparrow}(x)-f_i^{\downarrow}(x)\right]$

Ellis-Jaffe sum rule : $2\int_0^1 dx\, g_1(x) = C_1^{(3)} A_1$

Bjorken sum rule : $2\int_0^1 dx\, \left[g_1^p(x) - g_1^n(x)\right] = \frac{1}{6} C_1^{(3)} g_A$

Burkhardt-Cottingham sum rule : $\int_0^1 dx\, g_2(x) = 0$

**What is it like at strong coupling $\lambda$?**
(J.Polchinski and M.Strassler, JHEP0305:012,2003)

- Finite $x$ with $\lambda^{-1/2} << x < 1$, the unpolarized structure functions $F_1$ and $F_2$ are both power suppressed and vanish in the Bjorken limit.
- At small $x$, Graviton exchange contribution will survive in the large $q^2$ limit, which yields, $xF_1 \sim F_2 \propto x^{-1+\mathcal{O}(1/\sqrt{\lambda})}$, which give us a nonvanishing Momentum sum rule.

**What about the polarized structure functions at strong coupling ?**

# AdS/CFT prescription for current in DIS

- $\mathcal{R}$ current $J^{\mu}$ $\Leftrightarrow$ $A_m(x,z)$ Kaluza-Klein gauge field .
- $A_m(x,z)$ satisfy 5D Maxwell equation: $D_m F^{mn} = 0$
- In the Lorentz-like gauge : $\partial_\mu A^\mu + z\partial_z\left(A_z/z\right) = 0,$
- With the boundary conditions :

$$A_\mu(x,0) = A_\mu(x)|_{\rm 4d} = n_\mu e^{iq\cdot x} \qquad F_{z\mu}(y,z_0) = 0$$

- The nonnormalizable solutions are given by:

$$A_\mu = n_\mu e^{iq\cdot y} qz\left[\mathrm{K}_1(qz) + c\mathrm{I}_1(qz)\right]$$
$$A_z = in\cdot qe^{iq\cdot y} z\left[\mathrm{K}_0(qz) - c\mathrm{I}_0(qz)\right]$$

where $c = \mathrm{K}_0(qz_0)/\mathrm{I}_0(qz_0)$

*Break conformal symmetry by introducing a confinement scale $z_0$!*

# AdS/CFT prescription for "proton" in DIS

- "Proton" in " QCD" $\Leftrightarrow$ Charged Dilatino in "AdS".
- Dilatino field $\Psi(x,z)$ satisfy 5D Dirac equation:

$$(\not{D} - m)\Psi = 0$$

- Normalizable solution is given by:

$$\Psi(x,z) = Ce^{ip\cdot x} z^{\frac{5}{2}} \left[ J_{mR-1/2}(Mz)P_+ + J_{mR+1/2}(Mz)P_- \right] u_\sigma$$

where: $\not{p} u_\sigma = -iMu_\sigma (\sigma=1,2), \quad M^2 = -p^2, \quad P_\pm = \frac{1}{2}(1 \pm \gamma^5)$

- The interaction between $A_m$ and charged dilatino $\Psi$:

Minimal interaction : $S^M_{int} = iR^5 \int d^5x \sqrt{-g}\, \mathcal{Q} A_m e^m{}_a \overline{\Psi} \gamma^a \Psi$

# Calculate Structure functions from AdS

- Transition matrix element in "QCD" and "AdS"

$$\int d^4x \langle P_X, X, \sigma' | n_\mu J^\mu(x) | P, \mathcal{Q}, \sigma \rangle \quad \Leftrightarrow \quad i\mathcal{Q}R^5 \int d^4x dz \sqrt{-g}\, A_m \overline{\Psi}_X e^m{}_a \gamma^a \Psi_i$$

- The hadronic tensor is given by:

$$W_{\mu\nu} = (2\pi)^3 \sum_X \delta\Big(M_X^2 + (P+q)^2\Big) \mathcal{M}_\mu \mathcal{M}_\nu^*$$

- In the finite $x$ with $\lambda^{-1/2} << x < 1$ where supergravity approximation is valid, we can obtain both the unpolarized and polarized structure functions:

$$2F_1 = F_2 = 2g_1 = \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2}$$

$$2g_2 = \Big(\frac{1}{2x}\frac{\tau+1}{\tau-1} - \frac{\tau}{\tau-1}\Big) \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2}.$$

$$F_3 = g_3 = g_4 = g_5 = F_2 \qquad \text{(only linear } M/q \text{ kept !)}$$

# Discussion on the results at $\lambda^{-1/2} << x < 1$

- Similar to the unpolarized structure functions $F_1$ and $F_2$, the polarized structure functions $g_1$ and $g_2$, vanish with $(\Lambda^2/q^2)^{\tau-1}$ in the Bjorken limit $q \to \infty$, which is very different from QCD.
- In QCD, there's an interesting inequality $F_1 \geq g_1$, Here we see that $F_1 = g_1$ which indicates that the partons in the initial hadron is completely polarized.
- Typically there are just two different kinds of moments, e.g.,
  $$\int_0^1 2g_1(x,q^2)x^{n-1}\mathrm{d}x = \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} \frac{\Gamma(\tau-1)\Gamma(\tau+n+1)}{\Gamma(2\tau+n)}$$
  $$\int_0^1 2g_2(x,q^2)x^{n-1}\mathrm{d}x = \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} \frac{\Gamma(\tau-1)\Gamma(\tau+n)}{\Gamma(2\tau+n)} \frac{1-n}{2}.$$
  We expect that such results are correct at least for $n > 2$ where the low-$x$ contributions are negligible.
- Set $n = 1$ for $g_2$, one finds: $\int_0^1 \mathrm{d}x g_2(x,q^2) = 0$. which is completely independent of $\tau$ and $q^2$. In QCD, this sum rule is known as the Burkhardt-Cottingham sum rule.

# Polarized structure functions at the very small $x$

- Set n=1 for $g_1$ : $\int_0^1 2g_1(x,q^2)\mathrm{d}x = \pi A' \mathcal{Q}^2 (\Lambda^2/q^2)^{\tau-1} \frac{\Gamma(\tau-1)\Gamma(\tau+2)}{\Gamma(2\tau+1)}$.
  The integral vanishes as $q \to \infty$. This contradicts with the naive expectation that $\int_0^1 g_1\left(x,q^2\right)\mathrm{d}x$ should remain finite since the dilatino has spin-$\frac{1}{2}$.

**Where is spin at strong coupling ?**

(Y. Hatta et al JHEP,2009)

- At small x, string excitations and graviton exchange are the dominant contributions. $g_1$ arise from the t-channel exchange of a reggeized Kaluza-Klein photon:

$$g_1 \sim \frac{1}{x^{1-\mathcal{O}(\frac{1}{\sqrt{\lambda}})}}, \tag{1}$$

- There's no contribution for $g_2$ from such reggeized Kaluza-Klein photon, so the Burkhardt-Cottingham sum rule still holds.
- At strong coupling, Y.Hatta et al suggest the spin of a hadron entirely comes from the orbital angular momentum.

# Structure functions for the "neutron"

In the above, the interaction between the gauge field and charged dilatino is given by the minimal coupling

$$S_{int}^{M} = iR^5 \int d^5x\sqrt{-g}\, \mathcal{Q} A_m e^m{}_a \overline{\Psi}\gamma^a\Psi,$$

In order to consider the neutral dilatino, minimal coupling doesn't contribute. we need introduce the Pauli coupling:

$$S_{int}^{P} = \kappa R^6 \int d^5x\sqrt{-g}\, F_{mn} e^m{}_a e^n{}_b \overline{\Psi}\ [\gamma^a, \gamma^b]\Psi.$$

All the structure functions for the "neutron" are given by :

$$F_1^1 = g_1 = \frac{F_3}{2} = \frac{g_5}{2} = 16\pi\kappa^2 A'\left(\Lambda^2/q^2\right)^{\tau-1} x^{\tau+1}(1-x)^{\tau-2}[1-\tau(2-x)]^2,$$
$$F_2 = g_4 = 32\pi\kappa^2 A'\left(\Lambda^2/q^2\right)^{\tau-1} x^{\tau+1}(1-x)^{\tau-2}[1-\tau x(2-4\tau+3\tau x)],$$
$$g_2 = -8\pi\kappa^2 A'\left(\Lambda^2/q^2\right)^{\tau-1} x^{\tau}(1-x)^{\tau-2}\frac{1}{\tau-1}$$
$$\times\left[\tau(2\tau-5)+2\tau x(\tau^2-10\tau+8)+\tau x^2(7\tau^2+17\tau-6)-6\tau^2x^3(\tau+1)-1\right],$$
$$g_3 = 32\pi\kappa^2 A'\left(\Lambda^2/q^2\right)^{\tau-1} x^{\tau+1}(1-x)^{\tau-2}\frac{1}{\tau-1}\left[\tau(4x-3)-\tau^2(x^2+7\tau x^2-4x-6\tau x+2)-1\right].$$

# Comparisons between "proton" and "neutron" I

- The power order $\Lambda/q$ of the structure functions of the neutron from only Pauli interaction are the same as that from only minimal interaction.
- The relations $F_1^{\rm n} = \frac{F_3^{\rm n}}{2} = \frac{g_5^{\rm n}}{2}$ and $F_2^{\rm n} = g_4^{\rm n}$ still hold from the Pauli interaction, but the relation $F_1^{\rm p} = \frac{F_2^{\rm p}}{2} = \frac{g_3^{\rm p}}{2}$ from the minimal interaction is broken.
- The saturation condition $F_1 = g_1$ still holds, which indicates that the partons in the initial hadron is completely polarized.
- Burkhardt-Cottingham sum rule which holds for minimal interaction in the classic supergravity approximation is broken due to introducing the Pauli interaction term.

# Comparisons between "proton" and "neutron" II

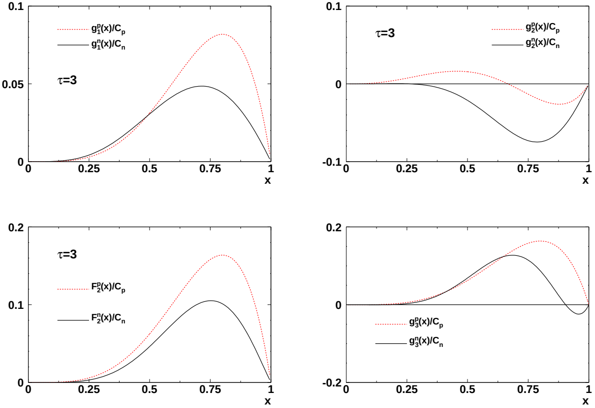
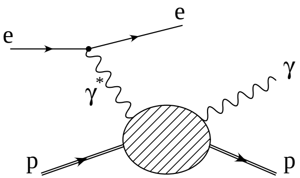


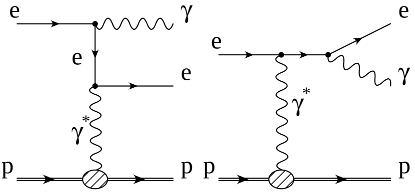

Figure: Illustration of the $g_1$, $g_2$, $F_2$ and $g_3$ for "proton" and "neutron". $C_{\rm p}=\frac{1}{2}\pi A' Q^2(\Lambda^2/q^2)^{\tau-1}$ and $C_{\rm n}=200\pi A'\kappa^2(\Lambda^2/q^2)^{\tau-1}$

# Introduction to DVCS

- The hadronic tensor can be expressed as the forward virtual Compton scattering amplitude by optic theorem :
$$W_{\mu\nu} = 4\pi \mathrm{Im}\, T_{\mu\nu}^{\gamma^* P \to \gamma^* P}.$$
- In order to extract more information for the hadron's structure, we can consider nonforward Compton scattering :
$$T_{\mu\nu}^{\gamma^* P \to \gamma^{(*)} P'} = i \int d^4 y e^{-iq\cdot y} \langle P' | \mathrm{T} J_\mu(y) J_\nu(0) | P \rangle.$$
- Study DVCS from $e^- + p \to e^- + p + \gamma$ process :

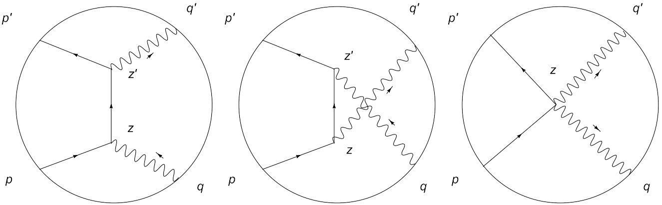



DVCS process.




BH process

# Witten diagram and bulk to bulk propagator for dilaton



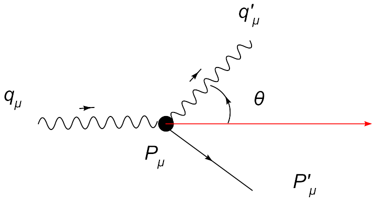

- Bulk to bulk propagator of dilatons in $AdS_5$ :

$$\left[z^2\partial_z^2-3z\partial_z+z^2\Box-\Delta(\Delta-4)\right]G(x,z;y,z')=z^5\delta(z-z')\delta^{(4)}(x-y).$$

The solution is given by:

$$G(x,z;y,z')=-\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}\int_0^\infty d\omega \frac{\omega}{\omega^2+k^2-i\epsilon}z^2J_{\Delta-2}(\omega z)z'^2J_{\Delta-2}(\omega z'),$$

# Some kinetic definitions




$$
\begin{aligned}
\text{Before the scattering} \quad p_\mu &= (M, 0, 0, 0)\,, \\
q_\mu &= (q_0, 0, 0, q_3)\,; \\
\text{After the scattering} \quad p'_\mu &= \left(p'_0, p'_1, 0, p'_3\right)\,, \\
q'_\mu &= \left(\omega', \omega' \sin\theta, 0, \omega' \cos\theta\right)\,.
\end{aligned}
$$

$$W = \sqrt{(P+q)^2}$$

# Real Compton Scattering off scalar from AdS/CFT

- Real Compton scattering $q^2 = q'^2 = 0$,

$$\mathcal{M} = 2\mathcal{Q}^2 c_i c_f \varepsilon(q,\lambda) \cdot \varepsilon'(q',\lambda') B,$$

with $B = z_0^{-2} \int_0^{z_0} dz\, z J_{\Delta-2}^2(Mz) > 0$.

- The total cross section:

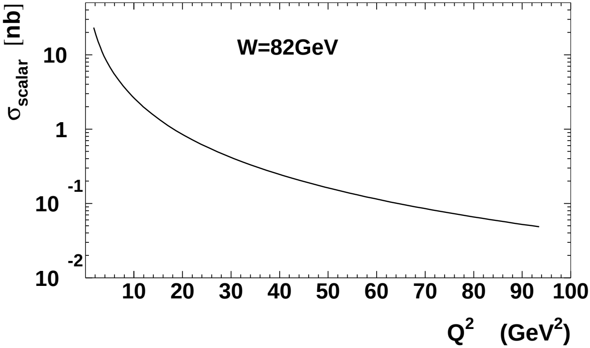
$$\frac{d\sigma}{d\Omega_{q'}} \propto \mathcal{Q}^4 \frac{q_0'^2}{q_0^2} \left(1 + \cos^2\theta\right)$$

- **Note** : It is identical to the cross section found for the elementary particle in scalar electrodynamics except for the overall constant.

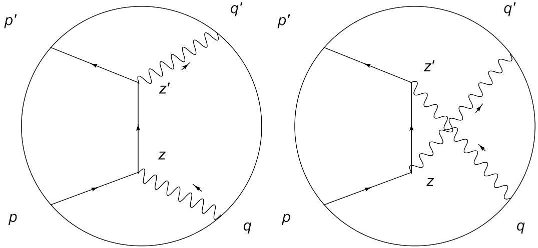
# DVCS off scalar from AdS/CFT

$$\frac{d\sigma}{d\cos\theta} = \frac{\omega'^2}{16M^2 q_0^2 (1-x)} \mathcal{Q}^4 \frac{c_i^2 c_f^2}{z_0^4} \mathcal{C}_{1\mathrm{DVCS}}^2 \left[1+\cos^2\theta+\chi\sin^2\theta\right],$$

with $\chi = \frac{q^2}{4M^2(1-x)^2}\left(1+\frac{4M^2}{q^2}x\right)^2$. When $q^2 \gg 4M^2$, $\chi \gg 1$.



# Witten diagram and bulk to bulk propagator for dilatino




Bulk to bulk propagator of dilatino in $A\mathrm{d}S_5$ :

$$\frac{z}{R}\left(\gamma^5\partial_z+\gamma^\mu\partial_\mu-\frac{2}{z}\gamma^5-\frac{mR}{z}\right)G(z,y;z',y')=\frac{z^5}{R}\delta(z-z')\delta^4(y-y').$$

The solution is given by:

$$G(y,z;y',z')=-\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(y-y')}\int_0^\infty d\omega\omega z^{5/2}\left[J_{mR-1/2}(\omega z)P_+ + J_{mR+1/2}(\omega z)P_-\right]$$
$$\times\frac{-i\not{k}+\omega}{\omega^2+k^2-i\epsilon}z'^{5/2}\left[J_{mR-1/2}(\omega z')P_- + J_{mR+1/2}(\omega z')P_+\right]$$

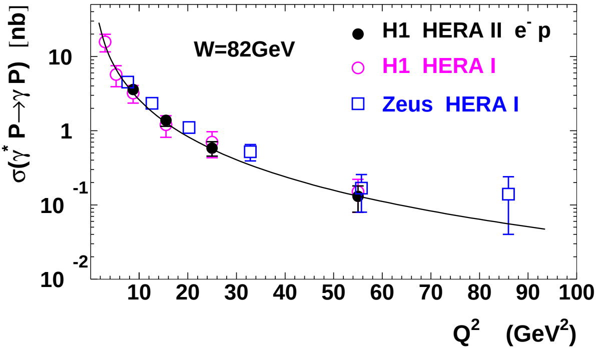
# Real Compton scattering off fermion from AdS/CFT

Setting both $q^2 = 0$ and $q'^2 = 0$, the real Compton scattering amplitude is found to be

$$\begin{aligned} T_{\mu\nu} &= -\mathcal{A}' z_0^2 J_{\tau-1}^2(Mz_0)\bar{u}_{\sigma'}(p')\gamma_\mu \frac{i(\not{p}+\not{q})+M}{M^2+(p+q)^2}\gamma_\nu u_\sigma(p) \\ &\quad -\mathcal{A}' z_0^2 J_{\tau-1}^2(Mz_0)\bar{u}_{\sigma'}(p')\gamma_\nu \frac{i(\not{p}-\not{q}')+M}{M^2+(p-q')^2}\gamma_\mu u_\sigma(p). \end{aligned}$$

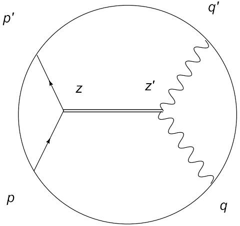
In such approximation, the real Compton scattering amplitude found above is parametrically the same as the Compton scattering amplitude of the fundamental particle of spin 1/2.

# Comparison with the experimental data




The DVCS ($\gamma^* p \to \gamma p$) cross section as a function of $Q^2$ for $W = 82 GeV$ and $|t| < 1 GeV^2$. The solid line represents the dilatino DVCS cross section after integrating over $t$. We set $\tau = 2$, the mass $M = 0.938 GeV$ to be the proton mass and $q^2 = Q^2$. We have used one of the data points to fix the overall constant.

# What is neglected in our calculations?




The $t$-channel graviton exchange amplitude can be written as:

$$\mathcal{A}^G = n^\mu T^G_{\mu\nu} n'^{*\nu} = \kappa^2 \int d^5y d^5y' T^{\Phi/\Psi}_{mn}(y) G^{mnkl}(y,y') T^A_{kl}(y'),$$

**The t-channel graviton exchange should be considered in the high energy limit!**

# Summary and Conclusion

- At strong coupling, the structure functions in the large $x$ region are all power suppressed and vanish in the Bjorken limit. All the dominant contributions are squeezed into small $x$ region. which is quite different from usual QCD.
- At strong coupling, Burkhardt-Cottingham sum rule holds for minimal interaction in the classic supergravity approximation, but is broken due to introducing the Pauli interaction term.
- At strong coupling, in the real Compton scattering process, the dilaton or dilatino act as an elementary particle. In Deeply inelastic scattering process, the results can fit the experiments very well on the dependence on the $Q^2$.
- The $t$-channel graviton exchange should dominate in the high energy limit and should be considered further in the future in order to compare to the experiments on the other variables dependence such as $s$ and $t$.

**Thank you!**