The pure annihilation type $B_c \to M_2 M_3$ decays in the perturbative QCD approach

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OUTLINE

1. Motivation

♠ In 1998, a new stage of B_c physics began with the first observation of the meson B_c at Tevatron. One can study the two heavy flavors b and c in B_c meson simultaneously.

♡ From an experimental point of view,

– The LHC experiment is now running, where the B_c meson could be produced abundantly.

– The B_c meson decays may provide windows for testing the predictions of the SM and can shed light on new physics scenarios beyond the SM.

♢ From a theoretical point of view,

– Due to its heavy-heavy nature and the participation of strong interaction, the non-leptonic decays of B_c meson complicate the extraction of parameters in SM;

– But, they provide great opportunities to study the perturbative and nonperturbative QCD, final state interactions, etc.;

– The non-leptonic B_c weak decays have been widely studied in literatures.

  * Naive factorization approach(NFA),

  * QCD factorization approach(QCDF),

  * Perturbative QCD approach(pQCD),

  * Other approaches and/or methods.

♠ Recently, charmless hadronic $B_c \to PP, PV/VP, VV$ decays have been studied [See Phys. Rev. D **80**-114031]. But, $Br(B_c \to \phi K^+, \overline{K}^{(*)0} K^{(*)+})_{\mathrm{SU(3)_F\ Symmetry}} \sim 10 \times Br(B_c \to \phi K^+, \overline{K}^{(*)0} K^{(*)+})_{\mathrm{QCDF}}$.

♣ The size of annihilation contributions is an important issue in B physics. For example, see Refs.[Eur. Phys. J. C **28**-305; ECONF **001**-C 070512; Phys. Lett. B **504**-6,**601**-151,**622**-63; Phys. Rev. D **63**-054008, **63**-074009,**71**-054025;Sci. China G **49**-357].

♠ The importance of annihilation contributions has already been tested in the previous predictions by employing the pQCD approach.

– Branching ratios of pure annihilation $B \to D_s K$ decays;

– Direct CP asymmetries of $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays;

– Explanation of $B \to \phi K^*$ polarization problem.

♡ Motivated by the important sizable annihilation contributions and the large discrepancies between the predictions by $\mathrm{SU(3)_F}$ symmetry and those with QCDF for $B_c \to M_2 M_3$[1], we here focus on these pure annihilation type decays within the framework of pQCD approach.

[1] We will use M_2 and M_3 to denote the two final state light mesons respectively, unless otherwise stated.

2. The pQCD Approach and Perturbative Calculations

In hadronic B decays, the dominant TH-uncertainties come from the evaluation of the relevant HME. Here the factorization approaches being used play the key role.

2.1 The pQCD Factorization Approach

In pQCD approach, the decay amplitude of $B_c \to M_2 M_3$ decays can be written conceptually as the convolution,

$$\mathcal{A} \sim \int d^4k_1 d^4k_2 d^4k_3 \, \mathrm{Tr}\left[C(t)\Phi_{B_c}(k_1)\Phi_{M_2}(k_2)\Phi_{M_3}(k_3)H(k_1,k_2,k_3,t)\right], \quad (1)$$

$k_i(i=1,2,3)$: the momenta of quark in the related mesons; Tr: the trace over Dirac and color indices; $C(t)$: the Wilson coefficients; $H(k_1,k_2,k_3,t)$: the hard kernel and can be calculated perturbatively; Φ_M: the wave function of the meson M; t: the largest energy scale in hard function H.

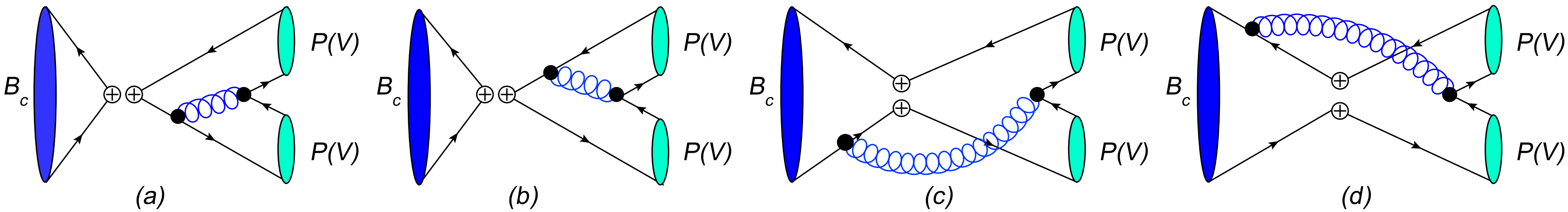

Figure 1: Typical Feynman diagrams contributing to the pure annihilation $B_c \to PP, PV/VP, VV$ decays at leading order.

♢ Choosing the light-cone coordinates: $P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, \mathbf{0}_T)$, $P_2 = \frac{m_{B_c}}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T)$, $P_3 = \frac{m_{B_c}}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T)$; with $r_2 = m_{M_2}/m_{B_c}$, and $r_3 = m_{M_3}/m_{B_c}$.

♣ the longitudinal polarization vectors, ϵ_2^L and ϵ_3^L, can be given by $\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2}m_{M_2}}(1 - r_3^2, -r_2^2, \mathbf{0}_T)$, $\epsilon_3^L = \frac{m_{B_c}}{\sqrt{2}m_{M_3}}(-r_3^2, 1 - r_2^2, \mathbf{0}_T)$. The transverse ones are parameterized as $\epsilon_2^T = (0, 0, 1_T)$, and $\epsilon_3^T = (0, 0, 1_T)$.

♡ Putting the (light-) quark momenta in B_c, M_2 and M_3 mesons as k_1, k_2, and k_3: $k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T})$, $k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T})$, $k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T})$.

The integration over k_1, k_2, and k_3 in Eq.(1) will lead to

$$\mathcal{A}(B_c \to M_2 M_3) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \mathrm{Tr}\left[C(t)\Phi_{B_c}(x_1, b_1)\times \Phi_{M_2}(x_2, b_2)\Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i)\, e^{-S(t)}\right]; \quad (2)$$

x_i: the momentum fraction of quark; b_i: the conjugate space coordinate of k_{iT}; $S_t(x_i)$: threshold resummation factor smearing the end-point singularities on x_i; $S(t)$: Sudakov form factor suppressing the soft dynamics effectively.

The weak effective Hamiltonian $H_{\rm eff}$ for $B_c \to M_2 M_3$ decays

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^* V_{uD} \left(C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)\right)\right] , \quad (3)$$

with the single tree operators,

$$\begin{aligned} O_1 &= \bar{u}_\beta \gamma^\mu (1-\gamma_5) D_\alpha \bar{c}_\beta \gamma^\mu (1-\gamma_5) b_\alpha \ , \\ O_2 &= \bar{u}_\beta \gamma^\mu (1-\gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \ , \end{aligned} \quad (4)$$

V_{ij}: the CKM matrix elements; Wolfenstein parametrization, $\lambda = 0.2257, A = 0.814, \bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$; $C_i(\mu)$: Wilson coefficients at the renormalization scale μ; "D": the light down quark d or s.

Wave functions for the related mesons,

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}} \left[(\not{P} + m_{B_c}) \gamma_5 \phi_{B_c}(x) \right]_{\alpha\beta} \ ; \quad (5)$$

$$\Phi_P(x) = \frac{i}{\sqrt{2N_c}} \gamma_5 \left\{ \not{P} \phi_P^A(x) + m_0^P \phi_P^P(x) + m_0^P (\not{n}\not{v} - 1) \phi_P^T(x) \right\}_{\alpha\beta} \ ; \quad (6)$$

$$\Phi_V^L(x) = \frac{1}{\sqrt{2N_c}}\left\{m_V \not\epsilon_V^{*L}\phi_V(x) + \not\epsilon_V^{*L} \not P \phi_V^t(x) + m_V\phi_V^s(x)\right\}_{\alpha\beta} ; \quad (7)$$

$$\Phi_V^T(x) = \frac{1}{\sqrt{2N_c}}\left\{m_V \not\epsilon_V^{*T}\phi_V^v(x) + \not\epsilon_V^{*T} \not P \phi_V^T(x) + m_V i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_T^{*\nu}n^\rho v^\sigma\phi_V^a(x)\right\}_{\alpha\beta} . \quad (8)$$

Note:

♣ Since B_c meson consists of two heavy quarks and $m_{B_c} \simeq m_b + m_c$, the distribution amplitude ϕ_{B_c} would be close to $\delta(x - m_c/m_{B_c})$ in the non-relativistic limit. We therefore adopt the non-relativistic approximation form of ϕ_{B_c} as,

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - m_c/m_{B_c}) , \quad (9)$$

where f_{B_c} and N_c are the decay constant of B_c meson and the color number.

♣ For pseudoscalar meson "$\eta - \eta'$" mixing, we adopt the quark-flavor basis as,

$$\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}, \qquad \eta_s = s\bar{s}. \tag{10}$$

The physical states η and η' are related to η_q and η_s through a single mixing angle ϕ,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi) \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \tag{11}$$

with

$$f_q = (1.07 \pm 0.02) f_\pi, \quad f_s = (1.34 \pm 0.06) f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ. \tag{12}$$

♣ For vector meson "$\omega - \phi$" mixing, we choose the ideal one, i.e., $\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}, \quad \phi = \bar{s}s$.

2.2 Perturbative Calculations

We firstly take the decays $B_c \to PP$ as an example to show the procedure of calculations in the pQCD approach. From the first two diagrams of Fig. 1, i.e., (a) and (b), by perturbative QCD calculations, one can obtain,

$$
\begin{aligned}
F_{fa}^{PP} &= -8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
&\times \left\{ h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a) \left[x_2 \phi_2^A(x_2) \phi_3^A(x_3) \right.\right. \\
&\left. + 2 r_0^2 r_0^3 \phi_3^P(x_3) \left((x_2+1) \phi_2^P(x_2) + (x_2-1) \phi_2^T(x_2) \right) \right] \\
&+ h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b) \left[(x_3-1) \phi_2^A(x_2) \phi_3^A(x_3) \right. \\
&\left.\left. + 2 r_0^2 r_0^3 \phi_2^P(x_2) \left((x_3-2) \phi_3^P(x_3) - x_3 \phi_3^T(x_3) \right) \right] \right\} \qquad (13)
\end{aligned}
$$

where $\phi_{2(3)}$ corresponding to the distribution amplitudes of mesons $M_{2(3)}$, $r_0^{2(3)} = m_0^{M_2(M_3)}/m_{B_c}$, and $C_F = 4/3$ is a color factor.

From the last two diagrams of Fig. 1, i.e., (c) and (d),

$$\begin{aligned}
M_{na}^{PP} &= -\frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \\
&\times \left\{ h_{na}^c(x_2,x_3,b_1,b_2) E_{na}(t_c) \left[(r_c - x_3 + 1)\phi_2^A(x_2)\phi_3^A(x_3) \right.\right. \\
&+ r_0^2 r_0^3 \left(\phi_2^P(x_2)((3r_c + x_2 - x_3 + 1)\phi_3^P(x_3) - (r_c - x_2 - x_3 + 1) \right. \\
&\times \phi_3^T(x_3)) + \phi_2^T(x_2)((r_c - x_2 - x_3 + 1)\phi_3^P(x_3) + (r_c - x_2 + x_3 - 1) \\
&\times \left.\left. \phi_3^T(x_3)))\right] - E_{na}(t_d) \left[(r_b + r_c + x_2 - 1)\phi_2^A(x_2)\phi_3^A(x_3) \right. \\
&+ r_0^2 r_0^3 \left(\phi_2^P(x_2)((4r_b + r_c + x_2 - x_3 - 1)\phi_3^P(x_3) - \phi_3^T(x_3) \right. \\
&\times (r_c + x_2 + x_3 - 1)) + \phi_2^T(x_2)((r_c + x_2 + x_3 - 1)\phi_3^P(x_3) \\
&- \left.\left.\left. (r_c + x_2 - x_3 - 1)\phi_3^T(x_3)))\right] h_{na}^d(x_2,x_3,b_1,b_2) \right\} , \qquad (14)
\end{aligned}$$

where $r_b = m_b/m_{B_c}$, $r_c = m_c/m_{B_c}$, and $r_b + r_c \approx 1$ for the B_c meson.

The general decay amplitude for $B_c \to M_2 M_3$ decays can be written as,

$$\mathcal{A}(B_c \to M_2 M_3) = V_{cb}^* V_{uD} \left\{ f_{B_c} F_{fa}^{M_2 M_3} a_1 + M_{na}^{M_2 M_3} C_1 \right\} , \quad (15)$$

where $F_{fa}^{M_2 M_3}(M_{na}^{M_2 M_3})$ come from the two factorizable(nonfactorizable) annihilation diagrams and $a_1 = C_1/3 + C_2$.

The decay amplitudes for $B_c \to \pi^+ \pi^0$ decays, for example, can be written as,

$$\mathcal{A}(B_c \to \pi^+ \pi^0) = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{fa}^{\pi^+ \pi^0_{\bar{u}u}} a_1 + M_{na}^{\pi^+ \pi^0_{\bar{u}u}} C_1] - [f_{B_c} F_{fa}^{\pi^0_{\bar{d}d} \pi^+} a_1 + M_{na}^{\pi^0_{\bar{d}d} \pi^+} C_1] \right\} , \quad (16)$$

The Feynman decay amplitudes for $B_c \to PV, VP$ can be got similarly.

There are three kinds of polarizations of a vector meson, namely, longitudinal (L), normal (N), and transverse (T).

The decay amplitudes $\mathcal{M}^{(\sigma)}$ in terms of helicities, for $B_c \to V(P_2, \epsilon_2^*)V(P_3, \epsilon_3^*)$ decays, can be generally described by

$$\mathcal{M}^{(\sigma)} \equiv m_{B_c}^2 \mathcal{M}_L + m_{B_c}^2 \mathcal{M}_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i\mathcal{M}_T \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{3\beta}^*(\sigma) P_{2\gamma} P_{3\rho} , \quad (17)$$

where the superscript σ denotes the helicity states of the two vector mesons with $L(T)$ standing for the longitudinal (transverse) component.

3. Numerical Results and Some Remarks

The masses (GeV), decay constants (GeV), QCD scale (GeV) and B_c meson lifetime to be used in the numerical calculations are as follows,

$$
\begin{aligned}
\Lambda_{\overline{\mathrm{MS}}}^{(f=4)} &= 0.250, \quad m_W = 80.41, \quad m_{B_c} = 6.286, \quad f_{B_c} = 0.489, \\
m_\phi &= 1.02, \quad f_\phi = 0.231, \quad f_\phi^T = 0.200, \quad m_{K^*} = 0.892, \\
f_{K^*} &= 0.217, \quad f_{K^*}^T = 0.185, \quad m_\rho = 0.770, \quad f_\rho = 0.209, \\
f_\rho^T &= 0.165, \quad m_\omega = 0.782, \quad f_\omega = 0.195, \quad f_\omega^T = 0.145, \\
m_0^\pi &= 1.4, \quad m_0^K = 1.6, \quad m_0^{\eta_q} = 1.08, \quad m_0^{\eta_s} = 1.92, \\
m_b &= 4.8, \quad f_\pi = 0.131, \quad f_K = 0.16, \quad \tau_{B_c} = 0.46\ ps\ . \qquad (18)
\end{aligned}
$$

The distribution amplitudes of light mesons P and V can be seen in Phys.Rev.D 81,014022 and references therein.

3.1 Numerical Results

In the following, we display the pQCD predictions of the branching ratios(BRs) for the considered $B_c \to M_2 M_3$ decays.

1. pQCD predictions of BRs for $B_c \to PP$ decays

♣ $\Delta S = 0$ processes(in unit of 10^{-7})

$$Br(B_c \to \pi^+\pi^0) = 0, \qquad Br(B_c \to \pi^+\eta) = 2.3^{+1.1}_{-0.8}\,; \quad (19)$$

$$Br(B_c \to \pi^+\eta') = 1.5^{+0.7}_{-0.5}, \quad Br(B_c \to K^+\overline{K}^0) = 2.4^{+1.0}_{-0.8}\,; \quad (20)$$

♣ $\Delta S = 1$ processes(in unit of 10^{-8})

$$Br(B_c \to \pi^+ K^0) = 4.0^{+2.6}_{-1.6}, \quad Br(B_c \to K^+\pi^0) = 2.0^{+1.3}_{-0.9}\,; \quad (21)$$

$$Br(B_c \to K^+\eta) = 0.6^{+0.6}_{-0.5}, \quad Br(B_c \to K^+\eta') = 5.7^{+1.3}_{-1.9}\,; \quad (22)$$

2. pQCD predictions of BRs for $B_c \to PV$ decays

♢ $\Delta S = 0$ processes(in unit of 10^{-7})

$$Br(B_c \to \pi^+\rho^0) = 1.7^{+0.6}_{-0.4}, \quad Br(B_c \to \pi^+\omega) = 5.8^{+1.8}_{-2.8}\,; \quad (23)$$

$$Br(B_c \to \overline{K}^0 K^{*+}) = 1.8^{+4.2}_{-2.1}\,; \quad (24)$$

♢ $\Delta S = 1$ processes(in unit of 10^{-8})

$$Br(B_c \to \rho^+ K^0) = 6.1^{+2.8}_{-3.3}, \quad Br(B_c \to K^+\rho^0) = 3.1^{+1.3}_{-1.7}\,; \quad (25)$$

$$Br(B_c \to K^+\omega) = 2.3^{+2.1}_{-1.2}\,; \quad (26)$$

3. pQCD predictions of BRs for $B_c \to VP$ decays

♡ $\Delta S = 0$ processes(in unit of 10^{-7})

$$Br(B_c \to \rho^+\pi^0) = 0.5^{+0.4}_{-0.4}, \quad Br(B_c \to \rho^+\eta) = 5.4^{+2.3}_{-1.8}\,; \quad (27)$$

$$Br(B_c \to \rho^+ \eta') = 3.6^{+1.5}_{-1.2}, \quad Br(B_c \to \overline{K^*}^0 K^+) = 10.0^{+1.8}_{-3.4}; \quad (28)$$

♡ $\Delta S = 1$ processes(in unit of 10^{-8})

$$Br(B_c \to \pi^+ K^{*0}) = 3.3^{+0.8}_{-0.6}, \quad Br(B_c \to K^{*+} \pi^0) = 1.6^{+0.5}_{-0.1}; \quad (29)$$

$$Br(B_c \to K^{*+} \eta) = 0.9^{+0.6}_{-0.2}, \quad Br(B_c \to K^{*+} \eta') = 3.8^{+1.5}_{-1.3}; \quad (30)$$

$$Br(B_c \to \phi K^+) = 5.6^{+1.7}_{-0.9}; \quad (31)$$

4. pQCD predictions of BRs for $B_c \to VV$ decays

♠ $\Delta S = 0$ processes(in unit of 10^{-6})

$$Br(B_c \to \rho^+ \rho^0) = 0.0; \quad (32)$$

$$Br(B_c \to \rho^+ \omega) = 1.1^{+0.4}_{-0.0} \quad (92.9^{+2.0}_{-0.1}\%); \quad (33)$$

$$Br(B_c \to \overline{K^*}^0 K^{*+}) = 1.0^{+0.8}_{-0.5} \quad (92.0^{+3.6}_{-7.1}\%); \quad (34)$$

♠ $\Delta S = 1$ processes(in unit of 10^{-7})

$$Br(B_c \to K^{*0}\rho^+) = 0.6^{+0.2}_{-0.1} \ (94.9^{+2.0}_{-1.4}\%), \tag{35}$$

$$Br(B_c \to K^{*+}\rho^0) = 0.3^{+0.1}_{-0.1} \ (94.9^{+2.0}_{-1.4}\%) \ ; \tag{36}$$

$$Br(B_c \to K^{*+}\omega) = 0.3^{+0.0}_{-0.2} \ (94.8^{+1.1}_{-1.2}\%), \tag{37}$$

$$Br(B_c \to K^{*+}\phi) = 0.5^{+0.1}_{-0.3} \ (86.4^{+4.9}_{-9.1}\%) \ . \tag{38}$$

3.2 Some Remarks

From our numerical evaluations and phenomenological analysis, we find the following results:

- Generally, CKM factor $|V_{ud}/V_{us}|^2 \sim 19 \Longrightarrow Br(B_c \to M_2M_3)_{\Delta S=0} > Br(B_c \to M_2M_3)_{\Delta S=1}$ as expected. Of course, for certain channels, this enhancement could be cancelled partly by the differences between the magnitude of individual decay amplitude.

- $Br(B_c \to \pi^+\pi^0, \rho^+\rho^0) = 0$; In fact, these two channels are forbidden, even if with final state interactions. Any other nonzero data for these two channels may indicate the effects of exotic new physics.

- Only tree operators $\Longrightarrow CP(B_c \to M_2M_3) = 0$.

- $Br(B_c \to M_2M_3)_{\rm pQCD} \in [10^{-8}, 10^{-6}]$; $Br(B_c \to \overline{K}^{*0}K^+, \overline{K}^{*0}K^{*+}, \rho^+\omega)_{\rm pQCD}$

$\sim 10^{-6}$ can be measured at the LHC experiment [Phys. Rev. D 80-114031].

Table 1: The pQCD predictions of branching ratios for $B_c \to \phi K^+$ and $B_c \to \overline{K}^{(*)0} K^{(*)+}$ modes. As a comparison, the numerical results as given in [Phys. Rev. D 80-114031] are also listed in the last two columns.

Channels	pQCD Predictions	$\mathrm{SU(3)_F}$ Symmetry	OGE model
$Br(B_c \to \phi K^+)$	$5.6^{+1.6}_{-0.9} \times 10^{-8}$	$\mathcal{O}(10^{-7} \sim 10^{-8})$	5×10^{-9}
$Br(B_c \to \overline{K}^0 K^+)$	$2.4^{+0.7}_{-0.6} \times 10^{-7}$	$\mathcal{O}(10^{-6})$	6.3×10^{-8}
$Br(B_c \to \overline{K}^0 K^{*+})$	$1.8^{+4.2}_{-2.1} \times 10^{-7}$	—	—
$Br(B_c \to \overline{K}^{*0} K^+)$	$1.0^{+0.2}_{-0.3} \times 10^{-6}$	$\mathcal{O}(10^{-6})$	9.0×10^{-8}
$Br(B_c \to \overline{K}^{*0} K^{*+})$	$1.0^{+0.8}_{-0.5} \times 10^{-6}$	$\mathcal{O}(10^{-6})$	9.1×10^{-8}

- In Table I, we find that $Br(B_c \to \phi K^+, \overline{K}^{*0} K^+$ and $\overline{K}^{*0} K^{*+})_{\mathrm{pQCD}} \approx Br(B_c \to \phi K^+, \overline{K}^{*0} K^+$ and $\overline{K}^{*0} K^{*+})_{\mathrm{SU(3)_F\ Symmetry}} \approx 10 \times Br(B_c \to$

$\phi K^+, \overline{K}^{*0} K^+$ and $\overline{K}^{*0} K^{*+})_{\rm OGE\ Model}$; $Br(B_c \to \overline{K}^0 K^+)_{\rm pQCD} < Br(B_c \to \overline{K}^0 K^+)_{\rm SU(3)_F\ Symmetry}$.

- The component $\bar{u}u + \bar{d}d$ contribute to the same decay amplitudes while the different mixing coefficients, i.e., $\cos\phi$ and $\sin\phi$ lead to the similar $Br(B_c \to (\pi^+, \rho^+)(\eta, \eta'))$.

- Rather different from the pattern of similar $Br(B_c \to (\pi^+, \rho^+)(\eta, \eta'))$, $Br(B_c \to K^+\eta') \sim 10 \times Br(B_c \to K^+\eta)$: opposite sign for η_q and η_s term, different coefficients $\Longrightarrow$ destruction for $Br(B_c \to K^+\eta)$ while construction for $Br(B_c \to K^+\eta')$, the similar pattern of $Br(B \to K\eta^{(\prime)})$.

- Factorizable contributions of η_s term $\Longrightarrow$ $Br(B_c \to K^{*+}\eta') \approx 4Br(B_c \to K^{*+}\eta) \sim 3.8 \times 10^{-8}$.

- $Br(B_c \to VV) \in [10^{-8}, 10^{-7}]$ except for $Br(B_c \to \overline{K}^{*0} K^{*+}, \rho^+\omega) \sim 10^{-6}$; $f_L(B_c \to VV) \sim 95\%$ within errors except for $f_L(B_c \to \phi K^{*+}) \sim 86\%$.

- Some simple relations in the limit of exact $\mathrm{SU}(3)_{\mathrm{F}}$ symmetry,

$$\mathcal{A}(B_c \to K^0\pi^+) = \sqrt{2}\mathcal{A}(B_c \to K^+\pi^0) = \lambda\mathcal{A}(B_c \to K^+\bar{K}^0)\,, \quad (39)$$

$$\mathcal{A}(B_c \to K^{*0}\pi^+) = \sqrt{2}\mathcal{A}(B_c \to K^{*+}\pi^0) = \lambda\mathcal{A}(B_c \to \bar{K}^{*0}K^+), \quad (40)$$

$$\mathcal{A}(B_c \to \rho^+K^0) = \sqrt{2}\mathcal{A}(B_c \to \rho^0K^+) = \lambda\mathcal{A}(B_c \to K^{*+}\bar{K}^0), \quad (41)$$

$$(-1)^\ell\mathcal{A}(B_c^+ \to \rho^+K^{*0}) = (-1)^\ell\sqrt{2}\mathcal{A}(B_c^+ \to \rho^0K^{*+}) = \lambda\mathcal{A}(B_c \to K^{*+}\bar{K}^0) \quad (42)$$

where $\lambda = V_{us}/V_{ud} \approx 0.2$ and $\ell = 0, 1, 2$.

- Frankly speaking, most $Br(B_c \to M_2M_3) \ll 10^{-6}$ still hard to observe even at LHC; Observation $\Longrightarrow$ large non-perturbative contribution or a signal for new physics beyond the SM.

- Sources of uncertainties: chiral mass m_0^P, values of Gegenbauer moments a_i and charm quark mass m_c, etc. Any progress in reducing the error will help us to improve the precision of the pQCD predictions.

4. Summary

- $Br(B_c \to M_2 M_3)_{\rm pQCD} \in [10^{-8}, 10^{-6}] \approx Br(B_c \to M_2 M_3)_{\rm SU(3)_F\ Symmetry}$;
- $V_{ud} \sim 1, V_{us} \sim 0.22 \Longrightarrow Br(B_c \to M_2 M_3)_{\Delta S=0} > Br(B_c \to M_2 M_3)_{\Delta S=1}$;
- Analogous to $B \to K\eta^{(\prime)}$ decays, $Br(B_c \to K^+\eta^\prime) \sim 10 \times Br(B_c \to K^+\eta)$;
- $f_L(B_c \to VV) \sim 95\%$ except for $f_L(B_c \to \phi K^{*+}) \sim 86\%$;
- Only tree operators $\Longrightarrow CP(B_c \to M_2 M_3) = 0$;
- large theoretical uncertainties from input parameters: m_0^P, a_i, m_c, etc.;
- possible long-distance contributions beyond the scope of this work.

Thanks For Your Attention!