Puzzles of Neutrino Mixing and Anti-Matter: Hidden Symmetries and Symmetry Breaking

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Common Origin of Soft $\mu-\tau$ and CP Breaking in Neutrino Seesaw and the Origin of Matter

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Abstract

Neutrino oscillation data strongly support μ–τ symmetry as a good approximate flavor symmetry of the neutrino sector, which has to appear in any viable theory for neutrino mass-generation. The μ–τ breaking is not only small, but also the source of Dirac CP-violation. We conjecture that both discrete μ–τ and CP symmetries are fundamental symmetries of the seesaw Lagrangian (respected by interaction terms), and they are *only softly broken, arising from a common origin via a unique*

Current Neutrino Oscillation Data

ν-Parameters	Lower Limit (2σ)	Best Value	Upper Limit (2σ)
Δm^2_{21} ($10^{-5}\mathrm{eV}^2$)	7.31	**7.67**	8.01
$\lvert\Delta m^2_{31}\rvert$ ($10^{-3}\mathrm{eV}^2$)	2.19	**2.39**	2.66
$\sin^2\theta_{12}$ (θ_{12})	0.278 (31.8°)	0.312 (34.0°)	0.352 (36.4°)
$\sin^2\boldsymbol{\theta_{23}}$ ($\boldsymbol{\theta_{23}}$)	0.366 (37.2°)	0.466 (**43.0**°)	0.602 (50.9°)
$\sin^2\boldsymbol{\theta_{13}}$ ($\boldsymbol{\theta_{13}}$)	0 (0°)	0.016 (**7.3**°)	0.036 (10.9°)

Evidence of $\mu-\tau$ Symmetry at Low Energy

- **Two small deviations (2σ level):**

$$-7.8^\circ < \theta_{23} - 45^\circ < 5.9^\circ \qquad 0 < \theta_{13} < 10.9^\circ$$

with **Best Fit Value**: $\boldsymbol{\theta_{23} - 45^\circ = -2.0^\circ}$ & $\boldsymbol{\theta_{13} = 7.3^\circ}$.

- **Zeroth Order Approximation:**

$$\boldsymbol{\theta_{23} = 45^\circ}, \qquad \boldsymbol{\theta_{13} = 0^\circ}.$$

with **Vanishing Dirac CP Phase** & $\mu-\tau$ **Symmetric Mass Matrix**:

$$M_\nu^{(0)} = \begin{pmatrix} A & \mathbf{B} & \mathbf{B} \\ & \mathbf{C} & D \\ & & \mathbf{C} \end{pmatrix}$$

Leptogenesis & Minimal Seesaw

- **Baryon Asymmetry ⇒ Leptogenesis ⇒ Seesaw**
- **Minimal Seesaw = SM + Two Heavy Majorana Neutrinos**

$$\mathcal{N}^T = (\mathcal{N}_\mu \quad \mathcal{N}_\tau)$$

- Lagrangian associated with Neutrino Masses:

$$\mathcal{L} = -\overline{L}_L Y_\ell \Phi \ell_R - \overline{\mathbf{L}}_\mathbf{L} \mathbf{Y}_\nu \widetilde{\mathbf{\Phi}} \mathcal{N} + \frac{\mathbf{1}}{\mathbf{2}} \mathcal{N}^\mathbf{T} \mathbf{M}_\mathbf{R} \mathcal{C} \mathcal{N}$$

$\mathbf{M_R}$ is Soft & High

- Dirac Mass Term:

$$m_D = Y_\nu \langle \widetilde{\Phi} \rangle$$

$\mathbf{\Phi}$ is the ordinary **SM Higgs Doublet**, **NO CP!**

Model Assignment at Zeroth Order

- **Minimal Seesaw** & $\mu - \tau$ & **CP Symmetries**:

$$T^{(3)}_{\mu\tau} = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix} \qquad T^{(2)}_{\mu\tau} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

- $\mathbf{T^{(3)}_{\mu\tau} m_D T^{(2)}_{\mu\tau} \equiv m_D}$ & $\mathbf{T^{(2)}_{\mu\tau} M_R T^{(2)}_{\mu\tau} \equiv M_R}$:

$$m_D = \begin{pmatrix} \mathbf{a} & \mathbf{a} \\ \mathbf{b} & \mathbf{c} \\ \mathbf{c} & \mathbf{b} \end{pmatrix}, \qquad M_R = \begin{pmatrix} \mathbf{m_{22}} & \mathbf{m_{23}} \\ \mathbf{m_{23}} & \mathbf{m_{33}} \end{pmatrix}$$

with all elements being **REAL**.

- **Seesaw Mass Matrix** for light neutrinos ($M_\pm \equiv m_{22} \pm m_{23}$):

$$\mathbf{M^{(0)}_\nu \approx m_D M_R^{-1} m_D^T} = \begin{pmatrix} \frac{2a^2}{M_+} & \frac{a(b+c)}{M_+} & \frac{a(b+c)}{M_+} \\ & \frac{1}{2}\left[\frac{(b+c)^2}{M_+} + \frac{(b-c)^2}{M_-}\right] & \frac{1}{2}\left[\frac{(b+c)^2}{M_+} - \frac{(b-c)^2}{M_-}\right] \\ & & \frac{1}{2}\left[\frac{(b+c)^2}{M_+} + \frac{(b-c)^2}{M_-}\right] \end{pmatrix}$$

Common $\mu - \tau$ & CP Soft Breaking

- **Approximate** $\mu - \tau$ symmetry @ **Zeroth-Order** $\Rightarrow$ **vanishing** θ_{13} & Dirac CP Phase δ_D;
- So, $\mu - \tau$ breaking should be **Small** & **Simultaneously** generates $\delta_D \Rightarrow \mu - \tau$ & **Dirac CP broken** by a **Common Origin**.
- **Natural** & **Simple**, so **Tempting**, to expect a **Common Origin** for all CP Phases;
- **Conjecture**: $\mu - \tau$ & **CP Symmetries** are **Softly broken** from a **Common Origin** which is **Uniquely** determined as:

$$M_R = m_{22} \begin{pmatrix} 1 & R \\ R & 1 - \zeta e^{i\omega} \end{pmatrix} \qquad \left(R \equiv \frac{m_{23}}{m_{22}} \right)$$

Note: $\mu - \tau$ & **CP Recover** with $\zeta \to \mathbf{0}$.

- **Hard Symmetry Breaking?** (Another paper in preparation)

Expected Consequences

- $\delta_a (\equiv \theta_{23} - 45^\circ)$ & $\delta_x (\equiv \theta_{13})$
  - **Common Origin** & **Linear** $\Rightarrow \delta_a \propto \delta_x$;
  - Once $\boldsymbol{\theta_{23}}$ well measured $\Rightarrow$ Predict $\boldsymbol{\theta_{13}}$!
- **Dirac CP Phase $\boldsymbol{\delta_D}$ & Majorana CP Phases**
  - **Common Origin** $\Rightarrow$ **Correlated**;
  - Once Dirac CP Phase $\boldsymbol{\delta_D}$ is measured $\Rightarrow$ **J** & $\mathbf{M_{ee}}$;
  - Vice Versa, Constrains from Leptogenesis.
- **Normal Hierarchy** with $\mathbf{m_1 = 0}$.
  - Fully reconstructed mass spectrum $\Rightarrow$ $\mathbf{M_{ee}}$;
  - Vice Versa.

Neutrino Mass Matrix from Seesaw

Expanding the mass matrix M_ν in terms of $\mathbf{r}$ & ζ up to **Linear Order**:

$$M_\nu = \mathbf{m_D M_R^{-1} m_D^T} \equiv \mathbf{M_\nu^{(0)}} + \mathbf{M_\nu^{(1)}} + \mathcal{O}(r^2, r\zeta, \zeta^2)$$

with:

$$\mathbf{M_\nu^{(0)}} = \frac{(\mathbf{b}-\mathbf{c})^2}{(2-\mathbf{X})\mathbf{M_1^{(0)}}} \begin{pmatrix} 0 & 0 & 0 \\ & 1 & -1 \\ & & 1 \end{pmatrix} \quad \text{with}$$

an Overall Phase
(**No Physical Consequence**)

$$\mathbf{M_\nu^{(1)}} = \frac{\mathbf{r}}{(2-\mathbf{X})^2\mathbf{M_1^{(0)}}} \begin{pmatrix} (2-\mathbf{X})^2\mathbf{a}^2 & (2-\mathbf{X})[(1-\mathbf{X})\mathbf{b}+\mathbf{c}]\mathbf{a} & (2-\mathbf{X})[\mathbf{b}+(1-\mathbf{X})\mathbf{c}]\mathbf{a} \\ & [(1-\mathbf{X})\mathbf{b}+\mathbf{c}]^2 & (1-\mathbf{X})(\mathbf{b}+\mathbf{c})^2+\mathbf{X}^2\mathbf{bc} \\ & & [\mathbf{b}+(1-\mathbf{X})\mathbf{c}]^2 \end{pmatrix} \equiv \begin{pmatrix} \delta m_{ee}^{(1)} & \delta m_{e\mu}^{(1)} & \delta m_{e\tau}^{(1)} \\ & \delta m_{\mu\mu}^{(1)} & \delta m_{\mu\tau}^{(1)} \\ & & \delta m_{\tau\tau}^{(1)} \end{pmatrix}$$

where $\mathbf{r} \equiv 1 - \mathbf{R}$ & $\mathbf{X} \equiv \frac{\zeta}{\mathbf{r}}\mathbf{e}^{\mathbf{i}\omega}$ and $\mathbf{M_1^{(0)}}$ is the **Zeroth-Order of the Lightest Eigenvalue** of $\mathbf{M_R}$:

$$\mathbf{M_1^{(0)}} = \mathbf{r}\,\mathbf{M_{22}}$$

Expanding Reconstructed Mass Matrix

$$\mathbf{M}_\nu = \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger \approx \mathbf{M}_\nu^{(0)} + \mathbf{M}_\nu^{(1)}$$

with:

$$M_\nu^{(0)} = \frac{1}{2} m_{30} \mathbf{e}^{-2\mathbf{i}\overline{\alpha}_{20}} \begin{pmatrix} 0 & 0 & 0 \\ & 1 & -1 \\ & & 1 \end{pmatrix}, \qquad M_\nu^{(1)} \equiv \begin{pmatrix} \delta m_{ee}^{(1)} & \delta m_{e\mu}^{(1)} & \delta m_{e\tau}^{(1)} \\ & \delta m_{\mu\mu}^{(1)} & \delta m_{\mu\tau}^{(1)} \\ & & \delta m_{\tau\tau}^{(1)} \end{pmatrix}$$

Note: **Overall CP Phase** **(No Physical Consequences!!!)**

For **Linear Order**:

$$\delta m_{ee}^{(1)} = m_{30} s_s^2 e^{-2i(\overline{\alpha}_{10} - \phi_{23})} y$$

$$\delta m_{\mu\mu}^{(1)} = \frac{1}{2} m_{30} e^{-2i\overline{\alpha}_{20}} \left[c_s^2 e^{-2i\phi_{23}} y + z + 2\delta_a - 2i\delta\overline{\alpha}_2 \right]$$

$$\delta m_{\tau\tau}^{(1)} = \frac{1}{2} m_{30} e^{-2i\overline{\alpha}_{20}} \left[c_s^2 e^{-2i\phi_{23}} y + z - 2\delta_a - 2i\delta\overline{\alpha}_3 \right]$$

$$\delta m_{e\mu}^{(1)} = \frac{1}{\sqrt{2}} m_{30} e^{-i(\overline{\alpha}_{10} + \overline{\alpha}_{20})} \left[-c_s s_s e^{-2i\phi_{23}} y + e^{-i\delta_D} \delta_x \right]$$

$$\delta m_{e\tau}^{(1)} = \frac{1}{\sqrt{2}} m_{30} e^{-i(\overline{\alpha}_{10} + \overline{\alpha}_{20})} \left[-c_s s_s e^{-2i\phi_{23}} y - e^{-i\delta_D} \delta_x \right]$$

$$\delta m_{\mu\tau}^{(1)} = \frac{1}{2} m_{30} e^{-2i\overline{\alpha}_{20}} \left[c_s^2 e^{-2i\phi_{23}} y - z + i(\delta\overline{\alpha}_2 + \delta\overline{\alpha}_3) \right]$$

Details

Solutions & Predictions

- **Zeroth-Order:**

  $$m_{10} = m_{20} = 0, \quad m_{30} = \frac{2(b-c)^2}{|(2-X)M_{10}|}, \quad e^{2i\overline{\alpha}_{30}} = \frac{r}{|r|}\frac{2-X}{|2-X|}$$

  Overall CP Phase (No Physical Consequence!)

- **Linear Order:**

  $$\delta_x = \frac{\sqrt{y} s_s \zeta}{2\left[\zeta^2 - 4r\zeta\cos\delta_D + 4r^2\right]^{1/4}}$$

  $$\delta_a = \frac{-\sqrt{y} c_s \cos\delta_D \zeta}{2\left[\zeta^2 - 4r\zeta\cos\delta_D + 4r^2\right]^{1/4}}$$

  - **Correlations:**

    $$\delta_x = -\frac{\tan\theta_s}{\cos\delta_D}\delta_a \qquad \Rightarrow \qquad |\delta_x| \geq \tan\theta_s |\delta_a|$$

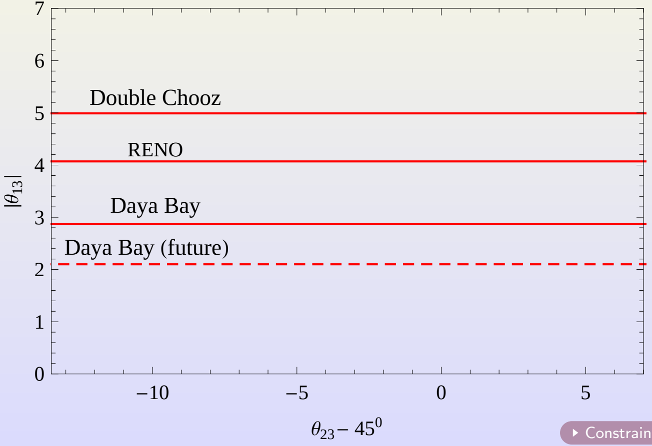
  - **Solar Mixing Angle θ_s Dictated by Dirac Mass Matrix m_D:**

    $$\tan\theta_s = -\frac{\sqrt{2}a}{b+c}$$

    Will be elaborated later.

Correlation between θ_{13} & θ_{23}

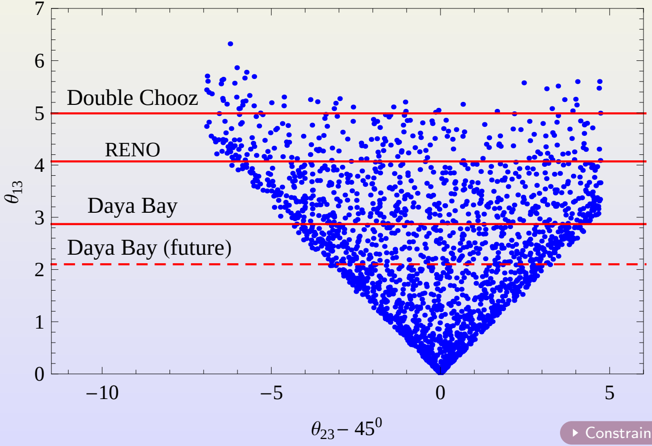
$$\delta_x = -\frac{\tan\theta_s}{\cos\delta_D}\delta_a$$



Constrained Correlation

Correlation between θ_{13} & θ_{23}

$$\delta_x = -\frac{\tan\theta_s}{\cos\delta_D}\delta_a$$



Constrained Correlation

Leptogenesis

- The Universe contains 4% Matter:
$$\eta_B \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10}$$
where n_γ is *Photon Number Density* & n_B is *Baryon Number Density*.

- Leptogenesis Mechanism generates η_B from *Lepton Asymmetry* Y_L via Sphaleron Interactions which violate $B + L$ but preserve $B - L$:
$$\eta_B = \frac{\xi}{f} N^f_{B-L} = -\frac{\xi}{f} N^f_L = -\frac{3\xi}{4f} \kappa_f \epsilon_f$$
where $\xi \equiv (8N_F + 4N_H)/(22N_F + 13N_H) = 28/79$ for SM, and $f = N^{rec}_\gamma / N^*_\gamma$ is the *Dilution Factor*.

- Efficiency Factor:
$$\kappa_f^{-1} \approx \left(\frac{\overline{m}_1}{0.55 \times 10^{-3} eV} \right)^{1.16} + \frac{3.3 \times 10^{-3} eV}{\overline{m}_1}$$
with $\overline{m}_1 \equiv (\widetilde{m}_D^\dagger \widetilde{m}_D)_{11} / M_1$ $(\widetilde{m}_D \equiv m_D V_R)$.

CP Asymmetry Parameter ϵ_1

- **CP Asymmetry Parameter ϵ_1:**

$$\epsilon_1 \equiv \frac{\Gamma[N_1 \to \ell H] - \Gamma[N_1 \to \overline{\ell} H^*]}{\Gamma[N_1 \to \ell H] + \Gamma[N_1 \to \overline{\ell} H^*]} = \frac{1}{4\pi v^2} F\left(\frac{M_2}{M_1}\right) \frac{\Im\mathfrak{m}\left\{\left[\left(\widetilde{m}_D^\dagger \widetilde{m}_D\right)_{12}\right]^2\right\}}{\left(\widetilde{m}_D^\dagger \widetilde{m}_D\right)_{11}}$$

Complex $\widetilde{m}_D$ differs $\Gamma[N_1 \to \ell H]$ from $\Gamma[N_1 \to \overline{\ell} H^*]$.

- **In Minimally Extended SM (Heavy Majorana Neutrinos):**

$$F(x) \equiv x\left[1 - (1+x^2)\ln\left(\frac{1+x^2}{x^2}\right) + \frac{1}{1-x^2}\right] = -\frac{3}{2x} + \mathcal{O}\left(\frac{1}{x^3}\right)$$

The expansion applies for $x \equiv M_2/M_1 \geq 5$.

- **In Current Model:**

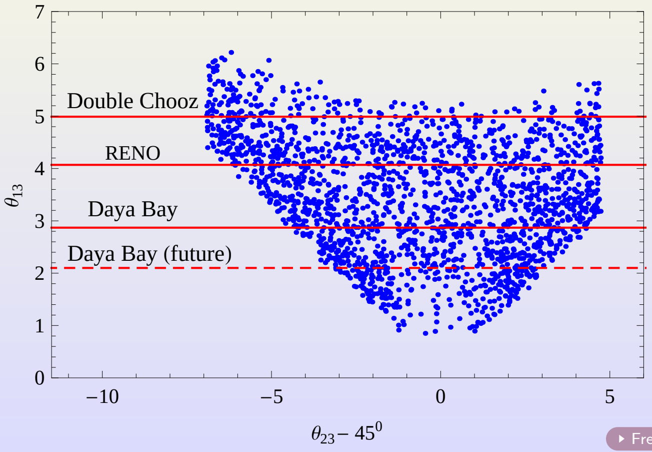
$$\epsilon_1 = -\frac{\widehat{m}_3 M_1}{4\pi v^2} \frac{3\left(4y - \sqrt{\zeta^2 - 4r\zeta\cos\delta_D + 4r^2}\right)^2}{128\left(\zeta^2 - 4r\zeta\cos\delta_D + 4r^2\right)} (4r\cos\delta_D - \zeta)\sin\delta_D\,\zeta^2$$

where $\widehat{m}_3$ is obtained by RG-running m_3 from M_Z to Leptogenesis Scale.

RGE

Lower Bound on θ_{13}

$$M_1 = \frac{4f}{3\xi}\frac{4\pi v^2}{\kappa_f \widehat{m}_3}\frac{128(4r^2 - 4r\zeta\cos\delta_D + \zeta^2)}{3\left[4y - \sqrt{4r^2 - 4r\zeta\cos\delta_D + \zeta^2}\right]^2}\frac{\eta_B}{(4r\cos\delta_D - \zeta)\sin\delta_D \zeta^2} \lesssim 10^{15}\,\text{GeV}$$



Free Correlation

θ_s Determined by $\mathbf{m_D}$

- As we have seen:

$$\tan\theta_s = -\frac{\sqrt{2}\mathbf{a}}{\mathbf{b}+\mathbf{c}}$$

which holds before and after soft breaking!

- Fully Determined by m_D:

$$\mathbf{m_D} = \begin{pmatrix} a & a \\ b & c \\ c & b \end{pmatrix}$$

- Not Affect by $\mu - \tau$ and CP symmetry breaking in M_R!

- **Protected or Accident?**

Extra $\mathbf{Z_2}$ Symmetry

- θ_s is **Solely** determined by $\mathbf{m_D}$;
- Soft symmetry breaking comes from M_R, m_D is not affect;
- If extra symmetry exists, it shouldn't be affected by soft breaking;
- It only applies on m_D, not M_R.

$$\mathbf{T_s^\dagger m_D = m_D}$$

- Can be realized by:

$$\nu_L \rightarrow \mathbf{T_s} \nu_L , \qquad \mathcal{N} \rightarrow \mathcal{N}$$

- Also respected by light neutrino's mass matrix M_ν:

$$\mathbf{T_s^T M_\nu T_s = M_\nu}$$

which is **Independent** of M_R.

Representation of the Extra Symmetry

- Neutrino mass matrix **Invariant** under transformation:

$$T_s^T M_\nu T_s = M_s$$

- Diagonalization Scheme:

$$V^T M_\nu V = D_\nu$$

- The effect of transformation is just a **Diagonal Rephasing**:

$$V^T T_s^T M_\nu T_s V = d_\nu D_\nu d_\nu = d_\nu V^T M_\nu V d_\nu$$

with $d_\nu^2 = I_3$ which constrains $d_\nu = \mathbf{diag}(\pm, \pm, \pm)$.

- General consequence:

$$T_s V = V d_\nu \quad \Rightarrow \quad T_s = V d_\nu V^\dagger$$

Representaion of the Extra Symmetry

- **Two Nontrivial Independent possibilities of d_ν:**

$$d_\nu^{(1)} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad d_\nu^{(2)} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}.$$

- **Mixing matrix with θ_s parameterized in terms of k:**

$$V(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{-\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **Two Independent symmetry transformations:**

$$T_s = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ 2k & k^2 & -2 \\ 2k & -2 & k^2 \end{pmatrix}, \qquad T_{\mu\tau} = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

$T_{\mu\tau}$ is 3D Representation of $\mu - \tau$ symmetry. $\mathbf{T}_{\mu\tau}^{(3)}$

Summary

- Oscillation Data strongly support $\mu - \tau$ symmetry as a Good Approximate Flavor Symmetry.
- The $\mu - \tau$ symmetry predicts $(\theta_{23}, \theta_{13}) = (45^\circ, 0^\circ)$ & Vanishing Dirac CP Phase.
- Conjecture: both $\mu - \tau$ and CP are Softly Broken by a Common Origin in M_R.
- With this conceptually Simple and Attractive construction, θ_{13} is Correlated with θ_{23} (Lower Bound on $|\delta_x|$ / Upper Bound on $|\delta_a|$).
  Strong supports for up-coming experiments.
- Predictions on Baryon Asymmetry through leptogenesis.
- Constrain by leptogenesis scale: Lower Bound on θ_{13}.
- Extra Z_2 dictating solar mixing angle θ_{12}.

Thank You!

Reconstruction of Light Neutrino Mass Matrix

Note: **Majorana Neutrino**'s mass matrix is **Symmetric**:

$$M_\nu \equiv V^* D_\nu V^\dagger = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ & m_{\mu\mu} & m_{\mu\tau} \\ & & m_{\tau\tau} \end{pmatrix} \quad \text{with} \quad D_\nu \equiv \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$$

where:

$$\begin{aligned} V &\equiv U'' U U', \\ U'' &\equiv \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}), \\ U' &\equiv \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}); \\ U &\equiv \begin{pmatrix} c_s c_x & -s_s c_x & -s_x e^{i\delta_D} \\ s_s c_a - c_s s_a s_x e^{-i\delta_D} & c_s c_a + s_s s_a s_x e^{-i\delta_D} & -s_a c_x \\ s_s s_a + c_s c_a s_x e^{-i\delta_D} & c_s s_a - s_s c_a s_x e^{-i\delta_d} & c_a c_x \end{pmatrix} \end{aligned}$$

$$(\theta_x \equiv \theta_{13}, \theta_s \equiv \theta_{12}, \theta_a \equiv \theta_{23})$$

Note: of the **Six Rephasing Phases**, only **Five** are **Independent**.

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Reconstructed Mass Matrix Elements

$$
\begin{aligned}
m_{ee} &= e^{-i2\alpha_1}\left[c_s^2 c_x^2 \widetilde{m}_1 + s_s^2 c_x^2 \widetilde{m}_2 + s_x^2 e^{-2i\delta_D}\widetilde{m}_3\right], \\
m_{\mu\mu} &= e^{-i2\alpha_2}\left[(s_s c_a - c_s s_a s_x e^{i\delta_D})^2 \widetilde{m}_1 + (c_s c_a + s_s s_a s_x e^{i\delta_D})^2 \widetilde{m}_2 + s_a^2 c_x^2 \widetilde{m}_3\right], \\
m_{\tau\tau} &= e^{-i2\alpha_3}\left[(s_s s_a + c_s c_a s_x e^{i\delta_D})^2 \widetilde{m}_1 + (c_s s_a - s_s c_a s_x e^{i\delta_D})^2 \widetilde{m}_2 + c_a^2 c_x^2 \widetilde{m}_3\right], \\
m_{e\mu} &= e^{-i(\alpha_1+\alpha_2)}\left[c_s c_x(s_s c_a - c_s s_a s_x e^{i\delta_D})\widetilde{m}_1 - s_s c_x(c_s c_a + s_s s_a s_x e^{i\delta_D})\widetilde{m}_2 \right. \\
&\qquad \left. + s_a s_x c_x e^{-i\delta_D}\widetilde{m}_3\right], \\
m_{e\tau} &= e^{-i(\alpha_1+\alpha_3)}\left[c_s c_x(s_s s_a + c_s c_a s_x e^{i\delta_D})\widetilde{m}_1 - s_s c_x(c_s s_a - s_s c_a s_x e^{i\delta_D})\widetilde{m}_2 \right. \\
&\qquad \left. - c_a s_x c_x e^{-i\delta_D}\widetilde{m}_3\right], \\
m_{\mu\tau} &= e^{-i(\alpha_2+\alpha_3)}\left[(s_s c_a - c_s s_a s_x e^{i\delta_D})(s_s s_a + c_s c_a s_x e^{i\delta_D})\widetilde{m}_1 \right. \\
&\qquad \left. + (c_s c_a + s_s s_a s_x e^{i\delta_D})(c_s s_a - s_s c_a s_x e^{i\delta_D})\widetilde{m}_2 - s_a c_a c_x^2 \widetilde{m}_3\right],
\end{aligned}
$$

with $\widetilde{\mathbf{m}}_{\mathbf{i}} \equiv \mathbf{m_i e^{-2i\phi_i}}$.

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Tiny Variables of Reconstructed Mass Matrix

- From:

$$M_\nu^{(0)} = \frac{(b-c)^2}{(2-X)M_1^{(0)}} \begin{pmatrix} 0 & 0 & 0 \\ & 1 & -1 \\ & & 1 \end{pmatrix}$$

we can get **Two Vanishing Mass Eigenvalues**:

$$m_1 = m_2 = 0$$

- **Normal Hierarchy** ⇒ **Nonzero** m_2:

$$y \equiv \frac{m_2}{m_3} \sim \mathcal{O}(r, \zeta)$$

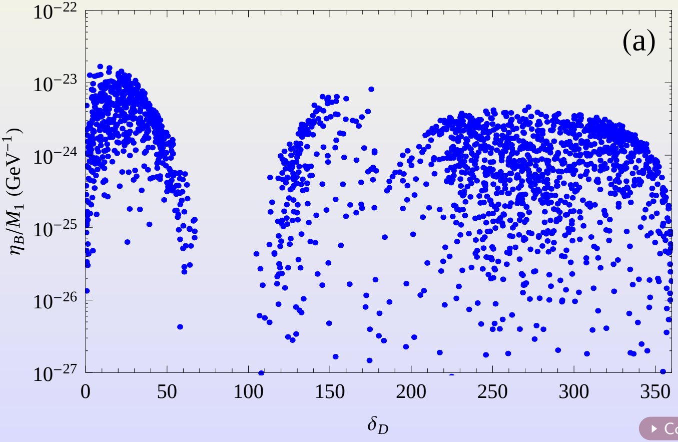
- Besides:

$$\delta_a, \delta_x, z \equiv \frac{\delta m_3}{m_3}, \delta\alpha_i \, (\overline{\alpha}_i \equiv \alpha_j + \phi_3)$$

Back

Prediction of Leptogenesis - η_B/M_1

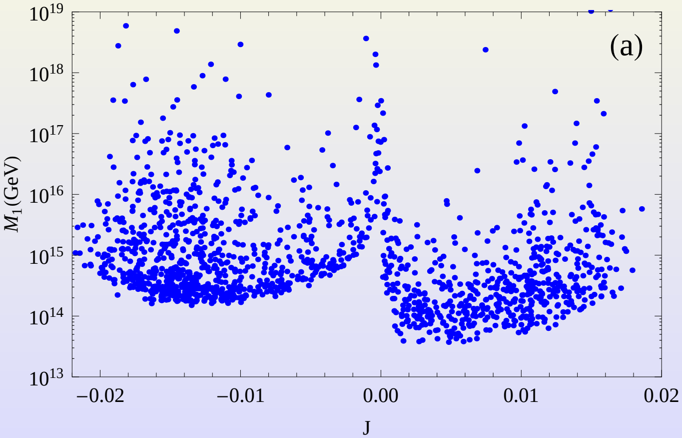
$$\frac{\eta_B}{M_1} = \frac{3\xi}{4f}\kappa_f \frac{\widehat{m}_3 M_1}{4\pi v^2} \frac{3\left(4y - \sqrt{\zeta^2 - 4r\zeta\cos\delta_D + 4r^2}\right)^2}{128\left(\zeta^2 - 4r\zeta\cos\delta_D + 4r^2\right)} (4r\cos\delta_D - \zeta)\sin\delta_D\,\zeta^2$$



Constrains on δ_D

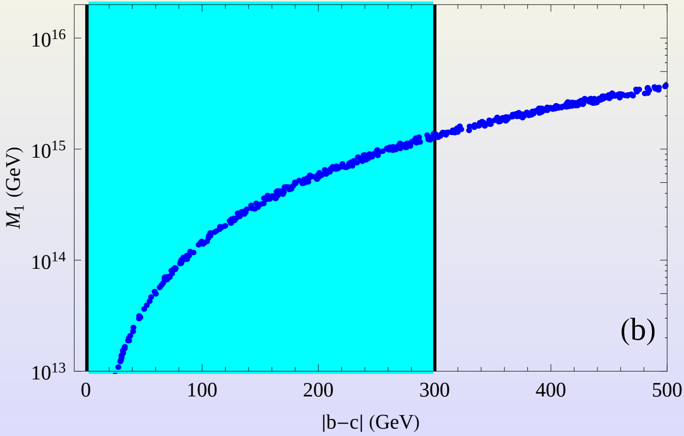
Prediction of Leptogenesis - Lower Limit of M_1

$$M_1 = \frac{4f}{3\xi}\frac{4\pi v^2}{\kappa_f \widehat{m}_3}\frac{128(4r^2 - 4r\zeta\cos\delta_D + \zeta^2)}{3\left[4y - \sqrt{4r^2 - 4r\zeta\cos\delta_D + \zeta^2}\right]^2}\frac{\eta_B}{(4r\cos\delta_D - \zeta)\sin\delta_D \zeta^2} \gtrsim 10^{13}\text{GeV}$$



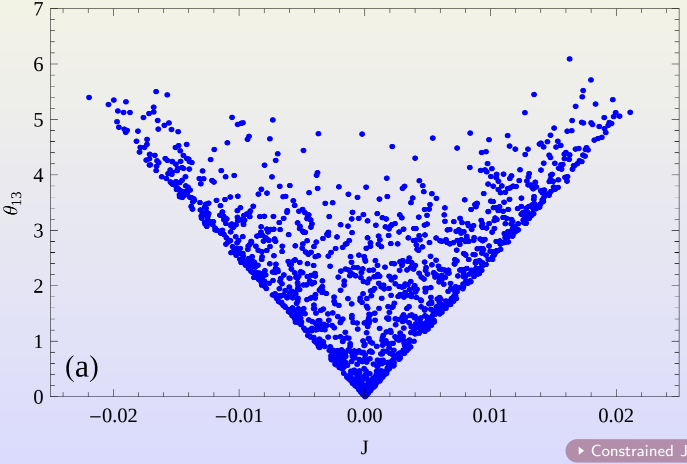
Upper Limit on Leptogenesis Scale M_1

$$M_1 = \frac{(b-c)^2}{\widehat{m}_3} \lesssim \mathbf{10^{15}}\text{GeV}$$



Jarlskog Invariant J

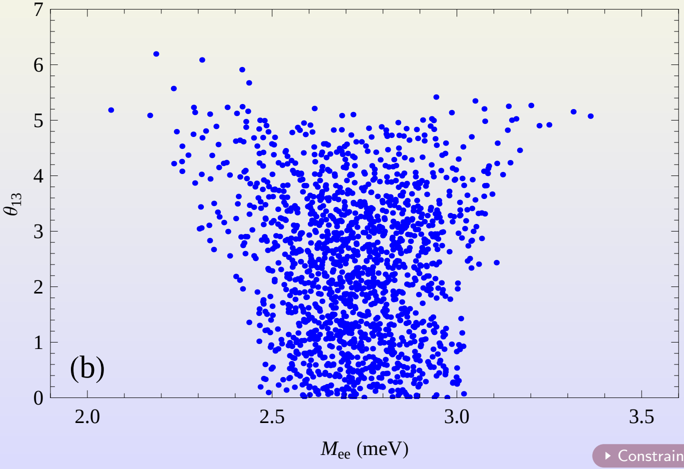
$$J = \frac{1}{4}\sin^2 2\theta_s \sin\delta_D \delta_x + \mathcal{O}\left(\delta_x^2, \delta_x\delta_a, \delta_a^2\right)$$



Constrained Jarlskog

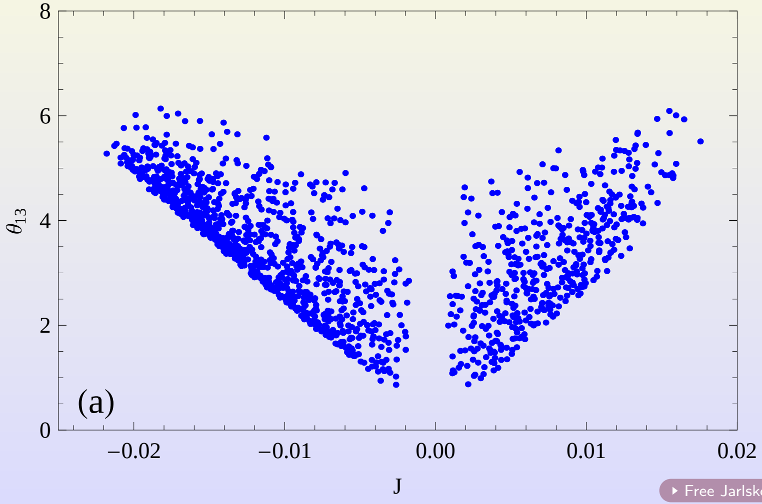
$0\nu 2\beta$ Decay Observable $|M_{ee}|$

$$M_{ee} \approx m_3 \sqrt{s_s^4 y^2 + 2 s_s^2 \cos 2(\delta_D - \phi_{23}) y \delta_x^2 + (\delta_x^4 - 2 s_s^2 y^2 \delta_x^2)}$$



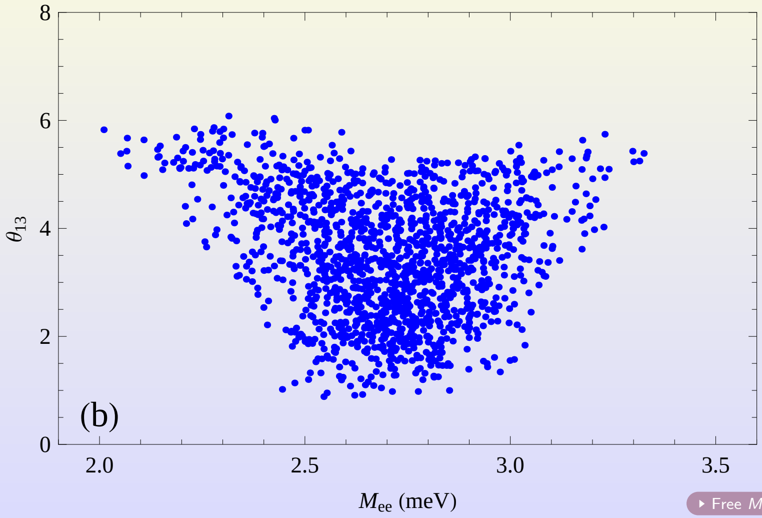
Constrained M_{ee}

Constrained Jarlskog Invariant J



Free Jarlskog

Constrained $0\nu 2\beta$ Decay Observable M_{ee}



▸ Free M_{ee}

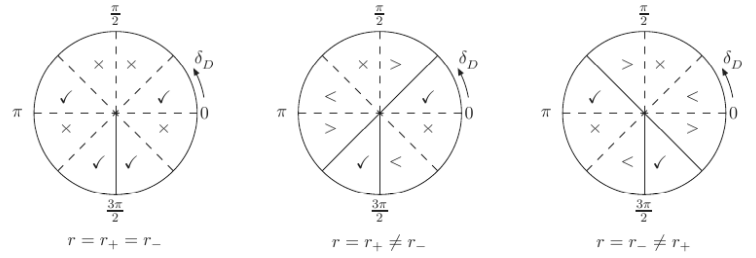
Constrained CP Phase δ_D by Leptogenesis

- $$\frac{\eta_B}{M_1} = \frac{3\xi}{4f}\kappa_f \frac{\widehat{m}_3}{4\pi v^2} \frac{3\left(4y - \sqrt{\zeta^2 - 4r\zeta\cos\delta_D + 4r^2}\right)^2}{128\left(\zeta^2 - 4r\zeta\cos\delta_D + 4r^2\right)} (4r\cos\delta_D - \zeta)\sin\delta_D\, \zeta^2$$

- $$\eta_B > 0 \quad \Rightarrow \quad (4r\cos\delta_D - \zeta)\sin\delta_D > 0$$

  $$r = \frac{\zeta}{2}\left[\cos\delta_D \pm \sqrt{\frac{s_s^4}{16}\frac{y^2\zeta^2}{\delta_x^4} - \sin^2\delta_D}\right] \quad \Rightarrow \quad \zeta \geq \frac{4}{s_s^2}\frac{\delta_x^2}{y}|\sin\delta_D|$$

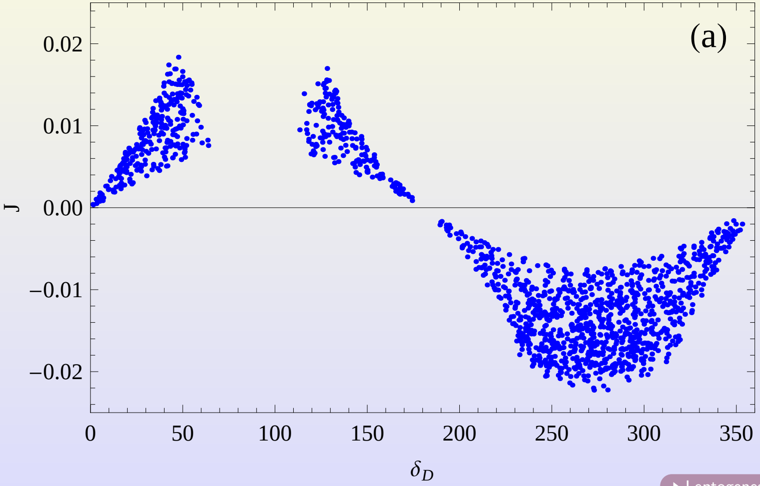
- These two inequalities will lead to:



$$\cos^2\delta_D \lessgtr \frac{4}{s_s^4}\frac{\delta_x^4}{y^2\zeta^2}$$

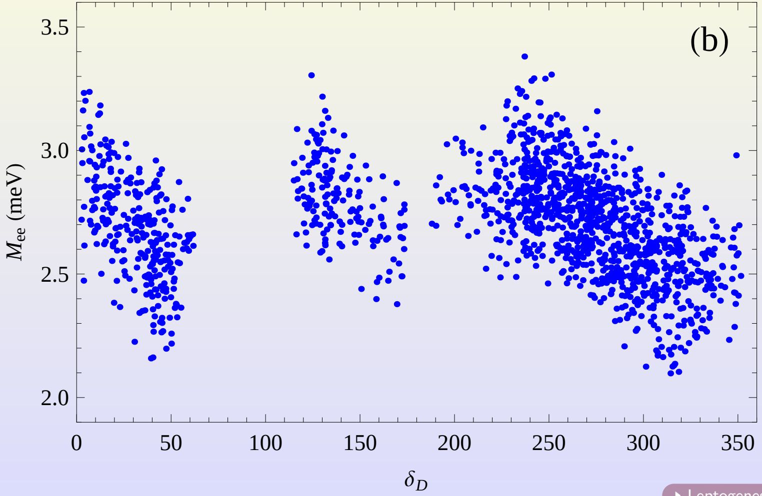
Leptogenesis

Constrained CP Phase δ_D v.s. J



Leptogenesis

Constrained CP Phase δ_D v.s. M_{ee}



Leptogenesis

RG Running Effect

- **Low Energy Observables $\overset{\text{RGE}}{\Longleftrightarrow}$ High Energy Observables**
- Only mass eigenvalues are obviously affected:

$$m_j(\mu) = \chi(\mu, \mu_0) m_j(\mu_0)$$

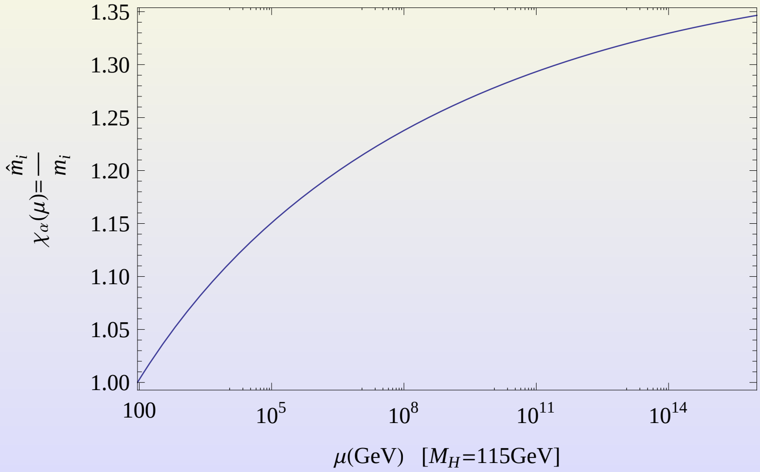
- which can be expressed as:

$$\chi(\mu, \mu_0) \approx \exp\left[\frac{1}{16\pi^2}\int_0^t \widehat{\alpha}(t')dt'\right] \quad \text{with} \quad \widehat{\alpha} \approx -2g_2^2 + 6y_t^2 + \lambda$$

- For leptogenesis: $\widehat{m}_j(M_1) = \chi(M_1, M_Z) m_j(M_Z)$

CP Asymmetry ϵ_1

RG Running Effect



CP Asymmetry ϵ_1