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Determination of chiral couplings at NLO in $1/N_C$: $L_8^r(\mu)$

J.J. Sanz-Cillero, 北京大學

(In collaboration with A. Pich & I. Rosell, Valencia U.)

[ArXiv: hep-ph/0610290]
[ArXiv: hep-ph/0610304]

Introduction and motivation

Recurrent problem in non-perturbative QCD:

→NEED FOR A QFT FOR MESONS (ρ, K^*, a_1, f_0,… D,B)

A way to afford the problem:

- **Large-N_C framework:**
  quark loops suppressed ⇔ meson loops suppressed

- **Perturbative calculation within a $1/N_C$ expansion:**
  Systematic contruction of a QFT for mesons

- **QCD long-distance ⇔Implementation of chiral symmetry:**
  We need a chiral theory for resonances (RχT)

- **Interconnection of low/high-energy QCD through RχT:**
  Predictions for the RχT and χPT coupling

Study of 2-point Green-functions

$$\Pi(t) = i\int dx^4 \; e^{iqx} \left\langle T\left\{ \bar{q}\Gamma q(x) \;\; \bar{q}\Gamma q(0)^\dagger \right\}\right\rangle$$, with $t=q^2$

We focus the attention on the SS-PP with I=1 [ArXiv: hep-ph/0610290, hep-ph/0610304]

- At low energies: χPT coupling L_8 (quark mass <-> pGoldstone mass)

$$\Pi(t) = B_0^2 \left[\frac{2\,F_\pi^2}{t} + 32\; L_8^r(\mu) + \frac{\Gamma_8}{\pi^2}\left(1 - \ln\frac{-t}{\mu^2}\right) + \mathcal{O}(t) \right]$$

PROGRAM:

1. Construction of a chiral lagrangian for resonances (**RχT**)

2. Calculation of **form-factors** at LO ← **Tree-level**

3. Derivation of $\Pi(t)$ at NLO (dispersive relations) ← **1-loop**

4. Matching OPE at NLO ← **1-loop**

5. Recovery of χPT at NLO ← **1-loop**

Reminder about the Leading Order in $1/N_C$

$$\Pi(t) = 2B_0^2 \left[\frac{F_\pi^2}{t} + \frac{8\,c_m^2}{M_S^2 - t} - \frac{8\,d_m^2}{M_P^2 - t} \right]$$

$$\Pi(t)\Big|^{R\chi T}_{N_C\to\infty} \xrightarrow{t\to-\infty} \Pi(t)^{OPE} = \sum_\ell \frac{\langle \mathcal{O}_{(2\ell)} \rangle}{(-t)^\ell}$$

$$\langle \mathcal{O}^{LR}_{(2)} \rangle = 0 \longrightarrow F_\pi^2 - 8\,c_m^2 + 8\,d_m^2 = 0$$

$$\langle \mathcal{O}^{LR}_{(4)} \rangle \simeq 0 \longrightarrow -8\,c_m^2\,M_S^2 + 8\,d_m^2\,M_P^2 \simeq 0$$

WSRs [Weinberg'67]

$$d_m^2 \simeq \frac{F^2}{8}\left(\frac{M_S^2}{M_P^2 - M_S^2}\right), \qquad c_m^2 \simeq \frac{F^2}{8}\left(\frac{M_P^2}{M_P^2 - M_S^2}\right)$$

$$\Pi(t) = B_0^2 \left[\frac{2F_\pi^2}{t} + 32 \left(\frac{F_\pi^2}{16}\left\{ \frac{1}{M_S^2} + \frac{1}{M_P^2} \right\} \right) + \mathcal{O}(t) \right]$$

$L_8^{N_C\to\infty}$ **At what μ ?**

Program

for calculations

at NLO in $1/N_C$

1.

Ingredients of a Resonance Chiral Theory (RχT)

[Ecker et al.'89], [Knecht & de Rafael'98] …

- Large N_C → $U(n_f)$ multiplets
- Goldstones from SχSB (π, K, η_8, η_0)
- **MHA:** First resonance multiplets **(V,A,S,P)**
- **Chiral symmetry** invariance
- Just $\mathcal{O}(p^2)$ operators

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{\chi PT}^{(2)} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1, R_2} \mathcal{L}_{R_1, R_2} + \sum_{R_1, R_2, R_3} \mathcal{L}_{R_1, R_2, R_3} + \dots$$

$$\mathcal{L}_{\chi PT}^{(2)} = \frac{F^2}{4}\langle u^\mu u_\mu + \chi_+ \rangle$$

$$\mathcal{L}_{\chi PT}^{(2)}$$

[Weinberg'79]

[Gasser & Leutwyler'84]

[Gasser & Leutwyler'85]

$$\mathcal{L}_V[\pi, V] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$\mathcal{L}_S[\pi, S]$, $\mathcal{L}_A[\pi, A]$, $\mathcal{L}_P[\pi, P]$

$$\sum_{R_1} \mathcal{L}_{R_1}$$

[Ecker et al.'89]

...

couplings λ_i^{RR}, λ_i^{RRR}

$$\sum_{R_1, R_2} \mathcal{L}_{R_1, R_2} + \sum_{R_1, R_2, R_3} \mathcal{L}_{R_1, R_2, R_3}$$

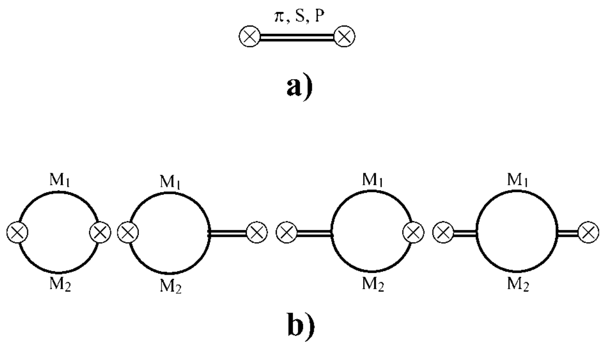
… [Moussallam'95], [Knecht & Nyffeler'01]

[Cirigliano et al.'06]

[Pich,Rosell & SC, forthcoming 06]

2. Form-factors at LO in $1/N_C$:

Spectral function at NLO



$$\mathrm{Im}\,\Pi(t)\big|_{\pi\pi} \sim \left|\mathcal{F}_{\pi\pi}(t)\right|^2 \xrightarrow{t\to\infty} 0$$ Vanishing Pion form factor

[Brodsky & Lepage'79]

This same behaviour is demanded to $\mathrm{Im}\,\Pi(t)\big|_{M_1,M_2}$

$$\mathrm{Im}\,\Pi(t)\big|_{M_1,M_2} \propto \left|\mathcal{F}_{M_1,M_2}(t)\right|^2 \xrightarrow{t\to\infty} 0$$

Stringent constraints on the **form-factors**

and resonance couplings λ_j

in terms of M_S, M_P, M_V, M_A

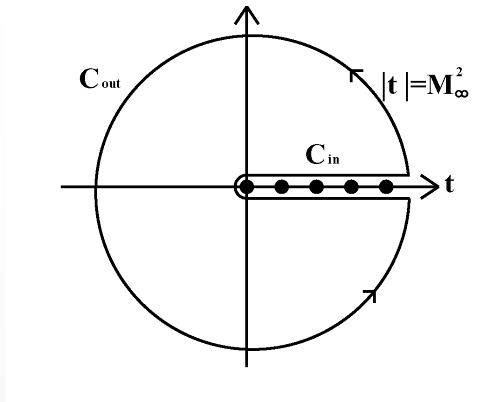
$$\mathcal{F}^{s}_{\pi\pi}(t) = \frac{M_S^2}{M_S^2 - t}$$

$$\mathcal{F}^{p}_{V\pi}(t) = -2\sqrt{1-\frac{M_V^2}{M_A^2}}\;\frac{M_V^2 M_P^2}{(M_P^2 - t)\,t}$$

...

3. Recontruction of Π(t) from Im Π(t) through an unsubtracted dispersion relation:

$$\Pi(t) = \frac{1}{2\pi i}\oint \frac{dt}{t'-t}\,\Pi(t') = \int_{\mathcal{R}^+} \frac{dt}{t'-t}\,\frac{1}{\pi}\,\mathrm{Im}\Pi(t')$$



At LO in $1/N_C$

$$\Pi(t) = 2B_0^2\left[\frac{F_\pi^2}{t} + \frac{8\,c_m^2}{M_S^2 - t} - \frac{8\,d_m^2}{M_P^2 - t}\right]$$

[Rosell, Pich & SC'04], [Rosell *et al.*'05]

(UV) Renormalized counter-terms — One-loop diagrams

At NLO in $1/N_C$

$$\Pi(t) = 2B_0^2\left[\frac{F^2}{t} + \frac{8\,c_m^{r\,2}}{M_S^{r\,2} - q^2} - \frac{8\,d_m^{r\,2}}{M_P^{r\,2} - q^2}\right] + \sum_{M_1,M_2} \Delta\Pi(t)\Big|_{M_1,M_2}$$

$$\left|\mathcal{F}_{M_1,M_2}(t)\right|^2$$

4. Matching RχT and OPE up to NLO in $1/N_C$:

- High energy limit of one-loop contributions:

$$\sum_{M_1,M_2} \Delta\Pi(t)\Big|_{M_1,M_2} \xrightarrow{t\to\infty} \frac{F^2}{t}(\delta_1) + \frac{F^2 M_S^2}{t^2}\left(\delta_2 + \tilde{\delta}_2 \ln\frac{-t}{M_S^2}\right) + O\left(\frac{1}{t^3}\right)$$

$f(M_S, M_P, M_V, M_A)$

- Matching the whole $\Pi(t)$ to OPE leads to

-No $\frac{1}{t^2}\ln\frac{-t}{M_S^2}$ term ➔ $\tilde{\delta}_2 = 0$

-1st and 2nd WSRs modified by terms NLO in $1/N_C$:

$$\langle \mathcal{O}_{(2)}^{LR} \rangle = 0 \longrightarrow F_\pi^2(1+\delta_1) - 8\, c_m^{r\,2} + 8\, d_m^{r\,2} = 0$$

$$\langle \mathcal{O}_{(4)}^{LR} \rangle \simeq 0 \longrightarrow -8\, c_m^{r\,2} M_S^{r\,2} + 8\, d_m^{r\,2} M_P^{r\,2} + F_\pi^2 M_S^2 \delta_2 = 0$$

$$d_m^{r\,2} = \frac{F^2}{8}\left(\frac{M_S^{r\,2}}{M_P^{r\,2} - M_S^{r\,2}}\right) \times [1 + \delta_1 - \delta_2], \; ...$$

5. Recovery of χPT at the one-loop level:

- Chiral symmetry ensures the right low-energy dynamics

(governed by χPT):

-Proper non-analytic ln(-t) structure

-Proper running of the LECs

$$\Pi(t)^{R\chi T} = B_0^2 \left[\frac{2F_\pi^2}{t} + 32\,\bar{L}_8 + \frac{\Gamma_8}{\pi^2}\left(1 - \ln\frac{-t}{M_S^2}\right) + \mathcal{O}(t) \right]$$

$\bar{L}_8$: $f(M_S^r, M_P^r, M_V)$

Prediction for $L_8^r(\mu)$ at any μ

E.g., for μ_o=770 MeV: $10^3 \cdot L_8^r(\mu_o) = 0.6 \pm 0.4$

Conclusions

- Chiral symmetric resonance lagrangian (RχT):

  Essential to recover low-energy QCD (χPT)

- 1/N_C expansion:

  Systematic expansion to compute loops with heavy particle

- Matching to high-energy QCD:

  Predictions for (renormalized) RχT and χPT

- **WE WANT A REAL QFT FOR MESONS**
  (not just narrow-width or Breit-Wigner ansate) !!!