New Members in the $0^{+}(0^{++})$ Family

刘　　翔

北京大学　物理学院

Xiao-Gang He, Xue-Qian Li, Xiang Liu and Xiao-Qiang Zeng, Phys. Rev. D73 (2006) 114026

Phys. Rev. D73 (2006) 051502(R)

高能物理学会第七届学术年会大会

桂林，2006年 10月28-11月1日

Motivation

- An enhancement has been observed by the BES collaboration in $J/\Psi \to \gamma\omega\phi$ radiative decay

$J^{PC}=0^{++}$

$X(1812)$: $M=1812^{+19}_{-26}(\text{state})\pm 18(\text{syst})\ \text{MeV/c}^2$

$\Gamma = 105 \pm 20(\text{stat}) \pm 28(\text{syst})\ \text{MeV/c}^2$

$$B(J/\Psi \to \gamma X(1812)) \bullet B(X(1812) \to \omega\phi)$$
$$=(2.61 \pm 0.27(\text{stat}) \pm 0.65(\text{syst})) \times 10^{-4}$$

- Recent BES data on $J/\Psi \to \phi\pi\pi$ indicate that there is a possible new spin-0 state

$f_0(1790)$: $M=1790^{+40}_{-30}\ \text{MeV/c}^2$ and $\Gamma = 270^{+60}_{-30}\ \text{MeV/c}^2$

$$B(J/\Psi \to \phi f_0(1790)) \bullet B(f_0(1790) \to \pi\pi) = (6.2 \pm 1.4) \times 10^{-4}$$

- In the energy range of 1 to 2 GeV, three $0^+(0^{++})$ states:

$f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

have been experimentally confirmed.

- Close et. al suggest that $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ may be mixtures of a scalar glueball G, an iso-singlet quarkonium $N=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $S=s\bar{s}$.

- $f_0(1790)$ + $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

G, N and S + Hybrid states

$$\xi_N = (u\bar{u}+d\bar{d})g/\sqrt{2}$$

$$\xi_S = s\bar{s}g/\sqrt{2}$$

New state X (1812)

The Studies on the Possible Structures of the Mixing

The mass matrix can be expressed as

$$M = \begin{pmatrix} M_{\xi_S} & 0 & e & 0 & 0 \\ 0 & M_{\xi_N} & \sqrt{2}e & 0 & 0 \\ e & \sqrt{2}e & M_G & f & \sqrt{2}f \\ 0 & 0 & f & M_S & 0 \\ 0 & 0 & \sqrt{2}f & 0 & M_N \end{pmatrix}$$

where $M_{\xi_S} = \langle \xi_S | \mathcal{H} | \xi_S \rangle$, $M_{\xi_N} = \langle \xi_N | \mathcal{H} | \xi_N \rangle$, $M_G = \langle G | \mathcal{H} | G \rangle$, $M_S = \langle S | \mathcal{H} | S \rangle$ and $M_N = \langle N | \mathcal{H} | N \rangle$ are the diagonal matrix elements of M.

$$e = \langle G | \mathcal{H} | \xi_S \rangle = \langle G | \mathcal{H} | \xi_N \rangle / \sqrt{2}.$$

$$f = \langle G | \mathcal{H} | S \rangle = \langle G | \mathcal{H} | N \rangle / \sqrt{2}$$

0

The matrix element <N|H|S> is obviously OZI suppressed

Diagonalizing the above matrix, one obtains the mass eigenvalues and physical states in terms of the quarkonia, hybrids and glueball. We parameterize the relation between the physical statesand the basis as

$$F_{phys} = U B_{basis}, \quad U = \begin{pmatrix} v_1 & w_1 & z_1 & y_1 & x_1 \\ v_2 & w_2 & z_2 & y_2 & x_2 \\ v_3 & w_3 & z_3 & y_3 & x_3 \\ v_4 & w_4 & z_4 & y_4 & x_4 \\ v_5 & w_5 & z_5 & y_5 & x_5 \end{pmatrix},$$

where $F^T_{phys} = (|X\rangle, |f_0(1790)\rangle, |f_0(1710)\rangle, |f_0(1500)\rangle, |f_0(1370)\rangle)$ and $B^T_{basis} = (|\xi_S\rangle, |\xi_N\rangle, |G\rangle, |S\rangle, |N\rangle)$. Here the state X is an extra 0^{++} state predicted in this scheme.

The mixing parameters w_i, v_i, z_i, y_i and x_i depend on the seven parameters

$M_{\xi_S,\xi_N,G,S,N}$, e and f

The available data which are directly related to these parameters are the five known eigenmasses of f_0(1790,1710,1500,1370) and X(1812)

To completely fix all the parameters, more information is needed

We use information from the ratios of the measured branching ratios of f_0(1790,1710,1500,1370) to two pseudoscalar mesons listed in Table 1.

The effective Hamiltonian of scalar state decaying into two pseudoscalar mesons can be written as [9]

$$\begin{aligned}\mathcal{H}_{eff}^{PP} &= f_1 \mathrm{Tr}[X_F P_F P_F] + f_2 X_G \mathrm{Tr}[P_F P_F] \\ &+ f_3 X_G \mathrm{Tr}[P_F]\mathrm{Tr}[P_F] + f_4 \mathrm{Tr}[X_H P_F P_F] \\ &+ f_5 \mathrm{Tr}[X_H P_F]\mathrm{Tr}[P_F] + f_6 \mathrm{Tr}[X_F]\mathrm{Tr}[P_F P_F] \\ &+ f_7 \mathrm{Tr}[X_F P_F]\mathrm{Tr}[P_F] + f_8 \mathrm{Tr}[X_F]\mathrm{Tr}[P_F]\mathrm{Tr}[P_F] \\ &+ f_9 \mathrm{Tr}[X_H]\mathrm{Tr}[P_F P_F] + f_{10} \mathrm{Tr}[X_H]\mathrm{Tr}[P_F]\mathrm{Tr}[P_F]. \end{aligned} \tag{3}$$

Here X_F is the flavor matrices of iso-singlet quarkonia components of X_i where the subscript $i = 1, ..., 5$ labels the five physical states. The detailed expression for X_F is given as [5]

$$X_F = a\lambda^0 + b\lambda^8 = \begin{pmatrix} \frac{u\bar{u}+d\bar{d}}{2} & 0 & 0 \\ 0 & \frac{u\bar{u}+d\bar{d}}{2} & 0 \\ 0 & 0 & s\bar{s} \end{pmatrix}$$
$$= \begin{pmatrix} \sum_i \frac{x_i}{\sqrt{2}} X_i & 0 & 0 \\ 0 & \sum_i \frac{x_i}{\sqrt{2}} X_i & 0 \\ 0 & 0 & \sum_i y_i X_i \end{pmatrix}$$

P_F is the pesudoscalar octet,

$$P_F = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{x_\eta \eta + x_{\eta'}\eta'}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{x_\eta \eta + x_{\eta'}\eta'}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & y_\eta \eta + y_{\eta'}\eta' \end{pmatrix}.$$

$x_{\eta,\eta'}$ and $y_{\eta,\eta'}$ describe the $\eta - \eta's$ mixing, and

$$x_\eta = y_{\eta'} = \frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{3}},$$
$$x_{\eta'} = -y_\eta = \frac{\sin\theta + \sqrt{2}\cos\theta}{\sqrt{3}},$$

The concrete expressions of X_G and X_H are

$$X_G = \sum_i z_i X_i,$$
$$X_H = (a\lambda^0 + b\lambda^8)g = \begin{pmatrix} \frac{u\bar{u}+d\bar{d}}{2}g & 0 & 0 \\ 0 & \frac{u\bar{u}+d\bar{d}}{2}g & 0 \\ 0 & 0 & s\bar{s}g \end{pmatrix}$$
$$= \begin{pmatrix} \sum_i \frac{w_i}{\sqrt{2}} X_i & 0 & 0 \\ 0 & \sum_i \frac{w_i}{\sqrt{2}} X_i & 0 \\ 0 & 0 & \sum_i v_i X_i \end{pmatrix}.$$

where $\theta = -19.1^\circ$[10] is the mixing angle of η and η'.

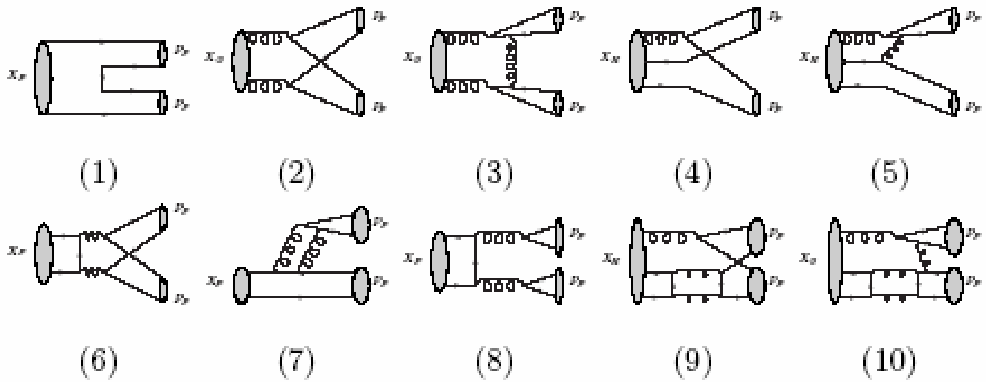

FIG. 1: The diagrams correspond respectively to terms in eq.(3). The last five terms are OZI suppressed ones.

$$
\begin{aligned}
\Gamma(X_i \to \pi\pi) &= 3 f_a^2 |\mathbf{p}_i| (x_i + \sqrt{2} z_i \xi_1 + w_i \xi_3)^2, \\
\Gamma(X_i \to K\bar{K}) &= f_a^2 |\mathbf{p}_i| (x_i + \sqrt{2} y_i + 2\sqrt{2} z_i \xi_1 + \sqrt{2} v_i \xi_3 + w_i \xi_3)^2, \\
\Gamma(X_i \to \eta\eta) &= f_a^2 |\mathbf{p}_i| [x_\eta^2 x_i + \sqrt{2} y_\eta^2 y_i + \sqrt{2}(x_\eta^2 + y_\eta^2) z_i \xi_1 \\
&\quad + (2\sqrt{2} x_\eta^2 + \sqrt{2} y_\eta^2 + 4 x_\eta y_\eta) z_i \xi_2 + \sqrt{2} y_\eta^2 v_i \xi_3 + x_\eta^2 w_i \xi_3 + (2 x_\eta^2 w_i \\
&\quad + \sqrt{2} y_\eta x_\eta w_i + 2 y_\eta x_\eta v_i + \sqrt{2} y_\eta^2 v_i) \xi_4]^2, \\
\Gamma(X_i \to \eta\eta') &= f_a^2 |\mathbf{p}_i| [\sqrt{2} x_\eta x_{\eta'} x_i + 2 y_\eta y_{\eta'} y_i + 2(x_\eta x_{\eta'} + y_\eta y_{\eta'}) z_i \xi_1 \\
&\quad + 2(2 x_\eta x_{\eta'} + \sqrt{2} x_\eta y_{\eta'} + \sqrt{2} x_{\eta'} y_\eta + y_\eta y_{\eta'}) z_i \xi_2 + 2 y_\eta y_{\eta'} v_i \xi_3 \\
&\quad + \sqrt{2} x_\eta x_{\eta'} w_i \xi_3 + (2\sqrt{2} x_\eta x_{\eta'} w_i + y_\eta x_{\eta'} w_i + y_{\eta'} x_\eta w_i + \sqrt{2} y_\eta x_{\eta'} v_i \\
&\quad + \sqrt{2} y_{\eta'} x_\eta v_i + 2 y_\eta y_{\eta'} v_i) \xi_4]^2,
\end{aligned}
$$

where $\xi_1 = f_2/f_1$, $\xi_2 = f_3/f_1$, $\xi_3 = f_4/f_1$ and $\xi_4 = f_5/f_1$ and $|\mathbf{p}_i|$ is the three-momentum of the final products in the center of mass frame of X_i,

$$
|\mathbf{p}_i| = \frac{\lambda^{1/2}(M_{X_i}^2, M_{P_1}^2, M_{P_2}^2)}{2M_{X_i}}, \tag{13}
$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. M_{X_i}, M_{P_1} and M_{P_2} are the mass of X_i and two final pseudoscalar mesons.

Numerical Results

* Five eigenmasses of $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$ and X(1812)

* Nine ratios of the branching ratios listed in Table 1

* Determine 11 parameters (7 parameters in the mass matrix+4 parameters ξ_i in the decay amplitudes)

We can carry out a χ^2 analysis with 3 degrees of freedom to test the QGH mixing mechanism.

	Experiment data[20]	Best fit
$\frac{\Gamma(f_0(1370)\to\pi\pi)}{\Gamma(f_0(1370)\to K\bar{K})}$	2.17 ± 0.90	2.22
$\frac{\Gamma(f_0(1370)\to\eta\eta)}{\Gamma(f_0(1370)\to K\bar{K})}$	0.35 ± 0.30	0.42
$\frac{\Gamma(f_0(1500)\to\pi\pi)}{\Gamma(f_0(1500)\to\eta\eta)}$	5.56 ± 0.93	5.45
$\frac{\Gamma(f_0(1500)\to K\bar{K})}{\Gamma(f_0(1500)\to\pi\pi)}$	0.33 ± 0.07	0.32
$\frac{\Gamma(f_0(1500)\to\eta\eta')}{\Gamma(f_0(1500)\to\eta\eta)}$	0.53 ± 0.23	0.26
$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710)\to K\bar{K})}$	0.20 ± 0.03	0.20
$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710)\to K\bar{K})}$	0.48 ± 0.19	0.27
$\frac{\Gamma(f_0(1710)\to\eta\eta')}{\Gamma(f_0(1710)\to K\bar{K})}$	< 0.04(90% C.L.)	0.007
$\frac{\Gamma(f_0(1790)\to\pi\pi)}{\Gamma(f_0(1790)\to K\bar{K})}$	$3.88^{+5.6}_{-1.9}$[2]	3.84
$M_{X(1812)}$(MeV)[1]	$1812^{+19}_{-26}(\mathrm{stat}) \pm 18(\mathrm{syst})$	1809
$M_{f_0(1790)}$(MeV)[2]	1790^{+40}_{-30}	1797
$M_{f_0(1710)}$(MeV)[15]	1714 ± 5	1714
$M_{f_0(1500)}$(MeV)[15]	1507 ± 5	1510
$M_{f_0(1370)}$(MeV)[15]	1350 ± 150	1242

Table 1: The measured and predicted central values for branching ratios and masses. The minimal χ^2 per degree of freedom is 1.26.

$$\begin{array}{ccccc} \xi_S & \xi_N & \mathrm{G} & \mathrm{S} & \mathrm{N} \end{array}$$

The best fit values for the mxing matrix elements are given by

$$U = \begin{pmatrix} -0.971 & -0.197 & -0.106 & -0.074 & -0.031 \\ -0.215 & +0.967 & +0.106 & +0.081 & +0.032 \\ -0.087 & -0.143 & +0.403 & +0.888 & +0.146 \\ +0.048 & +0.070 & -0.707 & +0.429 & -0.557 \\ +0.020 & +0.029 & -0.562 & +0.127 & +0.817 \end{pmatrix} . \begin{array}{l} X(1812) \\ f_0(1790) \\ f_0(1710) \\ f_0(1500) \\ f_0(1370) \end{array}$$

We see that the dominant component of $X(1812)$ is $s\bar{s}g$, whereas the $(u\bar{u} + d\bar{d})g/\sqrt{2}$ is the dominant one in $f_0(1790)$. The main components of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ are S, glueball(G) and N, respectively.

We see that the dominant component of $X(1812)$ is $s\bar{s}g$, whereas the $(u\bar{u}+d\bar{d})g/\sqrt{2}$ is the dominant one in $f_0(1790)$. The main components of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ are S, glueball(G) and N, respectively.

Parameter	Best fit and errors	Parameter	Best fit and errors
		e	20^{+8}_{-12} (MeV)
M_{ξ_S}	1807^{+58}_{-7} (MeV)	f	97^{+7}_{-6} (MeV)
M_{ξ_N}	1794^{+7}_{-23} (MeV)	ξ_1	$0.83^{+0.07}_{-0.03}$
M_G	1465^{+9}_{-9} (MeV)	ξ_2	$0.53^{+0.28}_{-0.37}$
M_S	1670^{+10}_{-11} (MeV)	ξ_3	$0.92^{+0.55}_{-0.73}$
M_N	1336^{+17}_{-10} (MeV)	ξ_4	$-3.08^{+3.41}_{-1.65}$

Table 2: The values for the parameters in the mass matrix M and the ratios $\xi_i = f_{1+i}/f_1$ $(i = 1 \sim 4)$ in the decay effective Hamiltonian $\mathcal{H}^{PP}_{eff}$.

QGH Predictions for $X(1812)$ and $f_0(1790)$ decays

$X(1812)(f_0(1790)) \to PP'$

$BR(X(1812) \to \pi\pi)$	4.4%
$BR(X(1812) \to K\bar{K})$	37.1%
$BR(X(1812) \to \eta\eta)$	32.6%
$BR(X(1812) \to \eta\eta')$	0.29%
$BR(f_0(1790) \to \pi\pi)$	16.8%
$BR(f_0(1790) \to K\bar{K})$	4.4%
$BR(f_0(1790) \to \eta\eta)$	9.8%
$BR(f_0(1790) \to \eta\eta')$	54.5%

Table 3: The central values for the branching ratios of $X(1812) \to PP'$ and $f_0(1790) \to PP'$.

$$X(1812)(f_0(1790)) \to VV'$$

$$\begin{aligned}
A(X_i \to \rho\rho) &= \sqrt{3}\Big(\tilde{g}_1 x_i + \sqrt{2}\tilde{g}_2 z_i + \tilde{g}_4 w_i\Big),\\
A(X_i \to K^*\bar{K}^*) &= \Big(\tilde{g}_1 x_i + \tilde{g}_1\sqrt{2}y_i + 2\sqrt{2}\tilde{g}_2 z_i + \tilde{g}_4\sqrt{2}v_i + \tilde{g}_4 w_i\Big),\\
A(X_i \to \omega\omega) &= \Big(\tilde{g}_1 x_i + \tilde{g}_2\sqrt{2}z_i + 2\sqrt{2}\tilde{g}_3 z_i + \tilde{g}_4 w_i + 2\tilde{g}_5 w_i\Big),\\
A(X_i \to \omega\phi) &= \Big(2\sqrt{2}\tilde{g}_3 z_i + \sqrt{2}\tilde{g}_5 v_i + \tilde{g}_5 w_i\Big),
\end{aligned}$$

where $\tilde{g}_j \approx g'_j \epsilon_{V_1} \cdot \epsilon_{V_2}$ with $g'_j = g_j(p_1 \cdot p_2 + a_j)$. Here we have only kept S-wave contribution since the decays are all close to the threshold and the dominant contribution comes from the S-wave term. With this approximation, there is just one parameter g'_j to consider for each of the terms.

$b_1 = \Gamma(X(1812) \to \rho\rho)/\Gamma(X(1812) \to \omega\phi)$, $b_2 = \Gamma(X(1812) \to \omega\omega)/\Gamma(X(1812) \to \omega\phi)$, $b_3 = \Gamma(X(1812) \to K^*\bar{K}^*)/\Gamma(X(1812) \to \omega\phi)$ and $b_4 = \Gamma(f_0(1790) \to \omega\omega)/\Gamma(b_0(1790) \to \rho\rho)$ depend on just one parameter $\beta = g'_5/g'_4$. In Figure 2, we show the ratios for b_i for β varying from 0.3 to 3 for illustration. We see that the relative branching ratios can change a large range. When more experimental data become available, information on the parameter β will be extracted.

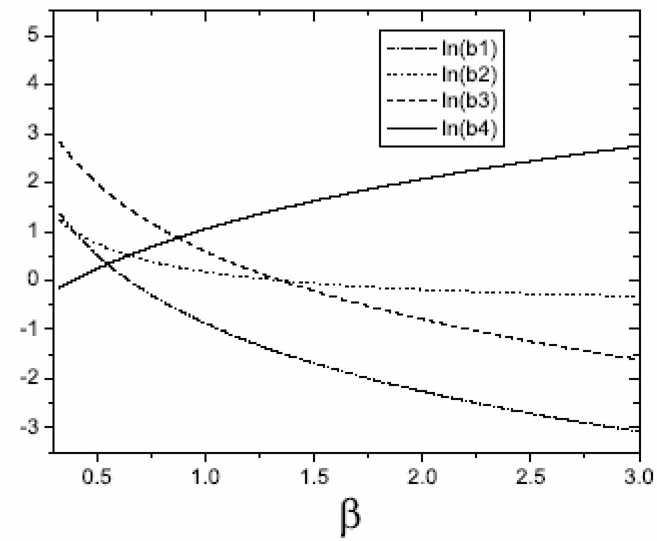

Figure 2: The dependence of b_i on the parameter of β.

Need more experimental information

$J/\psi \to \gamma X(1812)$

$$\langle\gamma X_i|J/\psi\rangle = x_i\langle\gamma N|J/\psi\rangle + y_i\langle\gamma S|J/\psi\rangle + z_i\langle\gamma G|J/\psi\rangle + w_i\langle\gamma\xi_N|J/\psi\rangle + v_i\langle\gamma\xi_S|J/\psi\rangle.$$

If the SU(3) symmetry applies, we would have

$$\langle\gamma S|J/\psi\rangle = \langle\gamma N|J/\psi\rangle/\sqrt{2} \quad \text{and} \quad \langle\gamma\xi_S|J/\psi\rangle = \langle\gamma\xi_N|J/\psi\rangle/\sqrt{2},$$

and the relations [21, 22] roughly hold

$$\langle\gamma G|J/\psi\rangle : \langle\gamma\xi_S|J/\psi\rangle : \langle\gamma S|J/\psi\rangle \sim 1 : \sqrt{\alpha_s} : \alpha_s.$$

We obtain an estimation

$$\Gamma(J/\psi \to \gamma X_i) = \frac{|\mathbf{k}_i|}{24\pi M^2_{J/\psi}}[\alpha_s(\sqrt{2}x_i + y_i) + \sqrt{\alpha_s}(v_i + \sqrt{2}w_i) + z_i]^2|M(J/\psi \to \gamma G)|^2,$$

where $\mathbf{k}_i$ is the three-momentum of final states in the center of mass frame of J/ψ.

$B(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}) = 8.5^{+1.2}_{-0.9}\times 10^{-4}$, $B(f_0(1710) \to K\bar{K}) = 0.38^{+0.09}_{-0.19}$

$|M(J/\psi \to \gamma G)|^2 = 0.005 \sim 0.016(0.007)\text{GeV}^2,$

$$\begin{aligned} B(J/\psi \to \gamma X(1812)) &= (0.3 \sim 1.0(0.4))\%, \\ B(X(1812) \to \omega\phi) &= (1.8 \sim 11.5(6.5))\%, \\ B(J/\psi \to \gamma f_0(1790)) &= (0.3 \sim 0.9(0.4))\%. \end{aligned}$$

Conclusion and discussion

- A mixing scheme : scalar quarkonia, scalar glueball and scalar hybrids

$0^{+}(0^{++})$: $X(1812)$ $f_0(1790)$ $f_0(1710)$ $f_0(1500)$ $f_0(1370)$

- $f_0(1710)$: $S = s\bar{s}$ $f_0(1790)$: $\xi_S = (u\bar{u}+d\bar{d})\,g$ $X(1812)$: $s\bar{s}g$

$f_0(1710)$:

- **Clear $f_0(1710)$** peak in $J/\psi \to \omega KK$.
- **No $f_0(1710)$** observed in $J/\psi \to \omega\pi\pi$!

$f_0(1790)$:

- Clear peak around 1790 MeV is observed in $J/\psi \to \phi\pi\pi$.

X(1812) :

Large branching ratio

$$B(J/\Psi \to \gamma X(1812)) \cdot B(X(1812) \to \omega\phi) = (2.61 \pm 0.27(stat) \pm 0.65(syst)) \times 10^{-4}$$

- Prediction:

$$X(1812) \rightarrow \pi\pi,\ KK,\ \eta\eta$$

$$f_0(1790) \rightarrow \pi\pi,\ KK,\ \eta\eta,\ \eta\eta'$$

$$J/\psi \rightarrow \gamma f_0(1790)$$

$$J/\psi \rightarrow \gamma X(1812)$$

BES and further experiments

Thanks!