Heavy-light transitions of baryons in QCD light-cone sum rules

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Motivation and Introduction

- Exclusive semileptonic decays of heavy hadrons are main source of information about CKM matrix elements and the field for understanding perturbative and nonperturbative QCD effects.

- The study of baryon sector is an important aspect of heavy flavor physics.
  - ⋆ Experimental drawback: the production rate of baryons is about an order of magnitude smaller than the rate for mesons. But next generation of experiments in hadron physics (e.g. LHCb) for more and more exclusive channels.
  - ⋆ Rather complicated internal structure of baryons compares to mesons.
  - ⋆ To consider the helicity of the quarks and spin structure by the decay of baryons.

- Exclusive semileptonic decays of heavy baryon include heavy-heavy transitions and heavy-light transitions. We focus on heavy-light transitions: $\Lambda_b \to p\ell\bar{\nu}$ and $\Lambda_c \to \Lambda\ell^+\nu$.

- The main difficulty is the calculation of hadronic matrix elements, which can be parametrized as form factors and are nonperturbative quantities.

- Nonperturbative QCD methods: lattice simulation, quark model, Bethe-Salpeter equation, HQET, QCD sum rules, ...

- Previous theoretical analysis for $\Lambda_b \to p\ell\bar{\nu}$ & $\Lambda_c \to \Lambda\ell^+\nu$

  ⋆ The quark model with heavy quark symmetry gives a result $\Gamma(\Lambda_b \to p\ell\bar{\nu}) = (6.48 - 7.47) \times 10^{12}|V_{ub}|^2 s^{-1}$.

  ⋆ The total decay width obtained from QCD sum rules in HQET is $\Gamma(\Lambda_b \to p\ell\bar{\nu}) = 1.35 \times 10^{-11}|V_{ub}|^2$GeV, while in full QCD this value is about $1.43 \times 10^{-11}|V_{ub}|^2$GeV.

  ⋆ The perturbative QCD factorization theorems prediction for $\Lambda_b \to p\ell\bar{\nu}$ is approximately $\Gamma = 1 \times 10^{-19}$GeV with $|V_{ub}| = 0.003$.

  ⋆ The non-relativistic quark model with flavor symmetry predicts $\Gamma(\Lambda_c \to \Lambda\ell^+\nu) = 7.1 \times 10^{10}s^{-1}$.

⋆ The relativistically covariant constituent quark model predicts $\Gamma(\Lambda_c \to \Lambda \ell^+ \nu) = 1.57 \times 10^{11} s^{-1}$.

⋆ The QCD sum rule method gives $\Gamma(\Lambda_c \to \Lambda \ell^+ \nu) = (8.7 \pm 1.2) \times 10^{-14}$GeV and the asymmetry parameter $\alpha = -1$.

- The existing theoretical predictions vary from each other and can differ even by orders of magnitude.
- Our theoretical method: Light-cone QCD sum rules

Main idea of the QCD (SVZ) sum rule

- The starting point is the correlation function of interpolating currents

$$\Pi(\omega)=i\int d^4x e^{ik\cdot x}\langle 0\mid T\{J_M^{+}(x),J_M(0)\}\mid 0\rangle.$$

- Admitting but not solving the asymptotic freedom of QCD Lagrangian, the method of operator product expansion(OPE) is employed in the actual calculation. Vacuum condensates are used to depict(characterize) the nonperturbative QCD vacuum.

- The theoretical side of the correlator can be calculated in the far Euclidean region and can be expressed as(up to irrelevant Lorentz structures): $\Pi(\omega)\approx\Pi_{pert}(\omega)+\Pi_{cond}(\omega)$, where the perturbative part can be written as dispersion integral

$$\Pi_{pert}(\omega)=\int d\nu\frac{\rho_{pert}(\omega)}{\nu-\omega-i\epsilon}+sub.,\ \Pi_{cond}(\omega)=\sum_n C_n\frac{\kappa}{(-\omega)^n}.$$

- While the phenomenological side has the form

$$\Pi_{hadr} = \sum_X \frac{\mid \langle X(v) \mid J_M \mid 0 \rangle \mid^2}{\omega_X - \omega - i\epsilon} + conti..$$

To obtain predicative information, the quark-hadron duality approximation is resorted to

$$\sum_{X \neq M} \frac{F_X^2(\mu)}{\omega_X - \omega - i\epsilon} \approx \int_{\omega_0}^{\infty} d\nu \frac{\rho_{pert}(\nu)}{\nu - \omega - i\epsilon}.$$

- The final sum rule looks like

$$\frac{F^2(\mu)}{2\bar{\Lambda} - \omega - i\epsilon} = \int_0^{\omega_0} \frac{\rho_{pert}}{\nu - \omega - i\epsilon} + sub. + \pi_{cond}(\omega).$$

- To improve the sum rule we need a Borel transformation

$$B_T^{\omega} \frac{1}{\nu - \omega - i\epsilon} = exp(-\nu/T), \qquad B_T^{\omega} \frac{1}{(-\omega)^n} = \frac{1}{\Gamma(n) T^{n-1}}$$

Light-cone sum rule

- The OPE breaks down at high momentum transfers.
- Contamination of the sum rule by ”nondiagonal” transitions of the ground state to excited states.
- Light-cone sum rules is a hybrid of the SVZ sum rules with the conventional distribution amplitude description of the hard exclusive process.
- Difference between SVZ sum rule and light-cone sum rule:
  - ⋆ T product inserted vacuum in the SVZ sum rules is replaced by the correlation function between a physical state and the vacua:

$$\int dx e^{-iqx}\langle 0|T\{J(0)j(x)\}|N(P)\rangle$$

  - ⋆ SVZ vacuum condensates in increasing dimension are substituted by light-cone hadron distribution amplitudes of increasing twist.

- The character of a hybrid of the standard QCD sum rule method with the conventional distribution amplitude description of hard exclusive process makes the light-cone QCD sum rule analysis suitable for decays with large momentum transfer.

- There have been numerous applications of light-cone sume rule to mesons. Such as: $B \to (\pi, \rho)\ell\bar{\nu}$, $B \to K^{(*)}\ell^+\ell^-$, $B \to \rho(\omega)\gamma$, $B \to K^{(*)}\gamma$, $B \to \pi\pi$, ...

Baryon distribution amplitudes

- Baryon distribution amplitudes are fundamental nonperturbative functions describing the hadron structure. The basic tool to describe DAs is provided by the conformal expansion.

- The definition for the nucleon DAs

$$
\begin{aligned}
&4\langle 0|\epsilon^{ijk}u^i_\alpha(a_1x)u^j_\beta(a_2x)d^k_\gamma(a_3x)|P\rangle \\
&= (\mathcal{A}_1 + \frac{x^2M^2}{4}\mathcal{A}_1^M)(\not{P}\gamma_5C)_{\alpha\beta}N_\gamma + \mathcal{A}_2M(\not{P}\gamma_5C)_{\alpha\beta}(\not{x}N)_\gamma \\
&+ \mathcal{A}_3M(\gamma_\mu\gamma_5C)_{\alpha\beta}(\gamma^\mu N)_\gamma + \mathcal{A}_4M^2(\not{x}\gamma_5C)_{\alpha\beta}N_\gamma \\
&+ \mathcal{A}_5M^2(\gamma_\mu\gamma_5C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu N)_\gamma + \mathcal{A}_6M^3(\not{x}\gamma_5C)_{\alpha\beta}(\not{x}N)_\gamma .
\end{aligned}
$$

For simplicity, only the axial vector DAs are presented for illustration.

- The calligraphic distribution amplitudes do not have definite twist, but can be related to the ones with definite twist as

$$\mathcal{A}_1 = A_1, \qquad 2P\cdot x\mathcal{A}_2 = -A_1 + A_2 - A_3,$$
$$2\mathcal{A}_3 = A_3, \qquad 4P\cdot x\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5,$$
$$4P\cdot x\mathcal{A}_5 = A_3 - A_4, \quad (2P\cdot x)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6.$$

- Each distribution amplitudes $F = V_i, A_i, T_i, S_i, P_i$ can be represented as

$$F(a_i p\cdot x) = \int \mathcal{D}x e^{-ip\cdot x\Sigma_i x_i a_i} F(x_i)\ .$$

The integration measure is defined as

$$\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1).$$

- Those distribution amplitudes are scale dependent and can be expanded in contributions of conformal operators. To the leading conformal spin accuracy the expansion reads

$$
\begin{aligned}
A_1(x_i,\mu) &= -120x_1x_2x_3\phi_3^0(\mu), \\
A_2(x_i,\mu) &= -24x_1x_2\phi_4^0(\mu), \\
A_3(x_i,\mu) &= -12x_3(1-x_3)\psi_4^0(\mu), \\
A_4(x_i,\mu) &= -3(1-x_3)\psi_5^0(\mu), \\
A_5(x_i,\mu) &= -6x_3\phi_5^0(\mu) \\
A_6(x_i,\mu) &= -2\phi_6^0(\mu),
\end{aligned}
$$

- At the case of Λ baryon, all the 6 parameters can be expressed in terms of 2 independent matrix elements of local operators:

$$
\phi_3^0=\phi_6^0=-f_\Lambda, \qquad \phi_4^0=\phi_5^0=-\frac{1}{2}\left(\lambda_1+f_\Lambda\right), \qquad \psi_4^0=\psi_5^0=-\frac{1}{2}\left(\lambda_1-f_\Lambda\right).
$$

- The coupling constant f_Λ and parameter λ_1 are defined by

$$\langle 0 \mid \epsilon_{ijk}[u^i(0) C\gamma_5 \not{z} d^j(0)] \not{z} s^k(0) \mid P\rangle = f_\Lambda z \cdot P \not{z} \Lambda(P),$$
$$\langle 0 \mid \epsilon_{ijk}[u^i(0) C\gamma_5 \gamma_\mu d^j(0)] \gamma^\mu s^k(0) \mid P\rangle = \lambda_1 M \Lambda(P).$$

- Most of the estimates of these nonperturbative parameters are obtained using QCD sum rules.

Light-cone sum rules for $\Lambda_b \to p\ell\bar{\nu}$

- The interpolating current for the Λ_b state is

$$j_\Lambda = \epsilon_{ijk}(u^i C \not{z} d^j)\gamma_5 \not{z} b^k,$$

where C is the charge conjugation matrix, and i, j, k denote the color indices. The auxiliary four-vector z, which satisfies the light-cone condition $z^2 = 0$, is introduced to project out the main contribution on the light-cone.

- The constant coupling the interpolating current to the vacuum is defined as

$$\langle 0 \mid j_\Lambda \mid \Lambda_b(P')\rangle = f_\Lambda z \cdot P' \not{z}\, u_\Lambda,$$

in which u_Λ is the Λ_b baryon spinor and P' is the Λ_b four-momentum.

- The correlation function we will consider in this work is

$$T_\nu(P, q) = i \int d^4x e^{iq\cdot x}\langle 0 \mid T\{j_\Lambda(0) j_\nu(x)\} \mid P\rangle$$

- The form factors are given by the following hadronic matrix element

$$\begin{aligned}\langle \Lambda_b(P-q) \mid j_\nu \mid P\rangle &= \bar{u}_\Lambda(P-q)\left[f_1\gamma_\nu - i\frac{f_2}{m_\Lambda}\sigma_{\nu\mu}q^\mu - \frac{f_3}{m_\Lambda}q_\nu\right.\\ &- \left.\left(g_1\gamma_\nu + i\frac{g_2}{m_\Lambda}\sigma_{\nu\mu}q^\mu + \frac{g_3}{m_\Lambda}q_\nu\right)\gamma_5\right]N(P),\end{aligned}$$

- The final sum rules are

$$\begin{aligned}&2f_\Lambda f_1 e^{-m_\Lambda^2/M_B^2} = \int_{x_0}^1 dx_2\, e^{-s'/M_B^2}\left\{B_0 + \frac{M^2}{M_B^2}\left[\frac{\mathcal{V}_1^{M(d)}(x_2)}{x_2} + \frac{m_b^2}{M_B^2}\frac{\mathcal{V}_1^{M(d)}(x_2)}{x_2^2}\right.\right.\\ &\left.\left.-B_1(x_2) - 2\,\frac{B_2(x_2)}{x_2} + 2\frac{Q^2}{M_B^2}\frac{B_2(x_2)}{x_2^2} + \frac{M^2}{M_B^2}B_3(x_2)\right]\right\} - \frac{M^2 e^{-s_0/M_B^2}}{m_b^2+Q^2+x_0^2M^2}\\ &\left[-x_0\mathcal{V}_1^{M(d)}(x_0) - \frac{m_b^2}{M_B^2}\mathcal{V}_1^{M(d)} + x_0^2B_1 + 2x_0B_2 - 2\frac{Q^2}{M_B^2}B_2 - \frac{M^2}{M_B^2}x_0^2B_3\right](x_0)\\ &-\frac{M^2e^{-s_0/M_B^2}x_0^2}{m_b^2+Q^2+x_0^2M^2}\frac{d}{dx_0}\left(\frac{m_b^2\mathcal{V}_1^{M(d)}(x_0) + 2Q^2B_2(x_0) + M^2x_0^2B_3(x_0)}{m_b^2+Q^2+x_0^2M^2}\right),\end{aligned}$$

and

$$
\begin{aligned}
&-\frac{2f_\Lambda f_2}{M m_\Lambda} e^{-m_\Lambda^2/M_B^2} = \frac{1}{M_B^2}\int_{x_0}^1 \frac{dx_2}{x_2}\, e^{-s'/M_B^2}\left(C_1(x_2) - \frac{M^2}{M_B^2} C_2(x_2)\right) \\
&+\frac{x_0 e^{-s_0/M_B^2}}{m_b^2 + Q^2 + x_0^2 M^2}\left(C_1(x_2) - \frac{M^2}{M_B^2} C_2(x_2)\right) \\
&+\frac{M^2 e^{-s_0/M_B^2} x_0^2}{m_b^2 + Q^2 + x_0^2 M^2}\frac{d}{dx}\left(\frac{x_0 C_2(x_0)}{m_b^2 + Q^2 + x_0^2 M^2}\right),
\end{aligned}
$$

where $s' = (1-x)M^2 + \frac{m_b^2 + (1-x)Q^2}{x}$, and x_0 is the positive solution of the quadratic equation for $s' = s_0$:

$$
2M^2 x_0 = \sqrt{(Q^2 + s_0 - M^2)^2 + 4M^2(Q^2 + m_b^2)} - (Q^2 + s_0 - M^2).
$$

The functions B_i and C_i are expressible via the DAs. Also we have $f_1 = g_1$ and $f_2 = g_2$.

LCSRs for the semileptonic $\Lambda_c \to \Lambda \ell^+ \nu$ decay

- The interpolating current for the Λ_c state is

$$j_{\Lambda_c} = \epsilon_{ijk}(u^i C\gamma_5 \not{z} d^j)\not{z} c^k.$$

The difference from that one interpolating Λ_b state is just a matter of convenience.

- The coupling constant and form factors are identically defined and the correlation function considered is the same.

- The final sum rules read

$$-f_{\Lambda_c} f_1 e^{-M_{\Lambda_c}^2/M_B^2} = -\int_{x_0}^1 dx_2\, e^{-s'/M_B^2}\left[B_0 + \frac{M^2}{M_B^2}\left(-B_1(x_3) + \frac{M^2}{M_B^2}B_2(x_3)\right)\right]$$
$$+\frac{M^2 x_0^2 e^{-s_0/M_B^2}}{m_c^2 + Q^2 + x_0^2 M^2}\left[B_1(x_0) - \frac{M^2}{M_B^2}x_0 B_2(x_0)\right]$$

$$+\frac{M^2 e^{-s_0/M_B^2} x_0^2}{m_c^2+Q^2+x_0^2M^2}\frac{d}{dx_0}\left(\frac{M^2x_0^2B_2(x_0)}{m_c^2+Q^2+x_0^2M^2}\right),$$

and

$$\frac{f_{\Lambda_c}f_2}{M_{\Lambda_c}M}e^{-M_{\Lambda_c}^2/M_B^2}=\frac{1}{M_B^2}\int_{x_0}^1\frac{dx_3}{x_3}\,e^{-s'/M_B^2}\left(B_3(x_3)-\frac{M^2}{M_B^2}B_2(x_3)\right)$$
$$+\frac{x_0e^{-s_0/M_B^2}}{m_c^2+Q^2+x_0^2M^2}\left(B_3(x_0)-\frac{M^2}{M_B^2}x_0B_2(x_0)\right)$$
$$+\frac{M^2e^{-s_0/M_B^2}x_0^2}{m_c^2+Q^2+x_0^2M^2}\frac{d}{dx_0}\left(\frac{x_0B_2(x_0)}{m_c^2+Q^2+x_0^2M^2}\right).$$

where

$$s'=(1-x)M^2+\frac{m_c^2+(1-x)Q^2}{x},$$

and x_0 is the positive solution of the quadratic equation for $s'=s_0$:

$$2M^2x_0=\sqrt{(Q^2+s_0-M^2)^2+4M^2(Q^2+m_c^2)}-(Q^2+s_0-M^2).$$

NUMERICAL ANALYSIS

- Parameters: The mass of the $\Lambda_b, \Lambda_c, \Lambda$ and p is taken from the PDG; the heavy quark mass adopted is $m_b = 4.8$GeV and $m_c = 1.41$GeV; sum rules for the various coupling constants are substituted into the light-cone sum rules to obtain numerical results.

- Numerical results for $\Lambda_b \to p\ell\bar{\nu}$
  - ⋆ Both form factors can be fitted well by the dipole formula

$$f_i(q^2) = \frac{f_i(0)}{a_2(q^2/m^2_{\Lambda_b})^2 + a_1 q^2/m^2_{\Lambda_b} + 1}.$$

  - ⋆ For the distribution amplitudes obtained from the the asymptotic and QCD sum rule methods, we give the coefficients for a special set of values:

	asymptotic			QCD sum rule		
	a_2	a_1	$f_i(0)$	a_2	a_1	$f_i(0)$
f_1	2.381	-2.888	-0.037	5.590	-2.759	0.018
f_2	2.582	-3.026	0.027	2.000	-2.603	0.159

- ⋆ Note that the light-cone expansion is expected to hold only at $q^2 \leq m_b^2 - O(\Lambda_{\rm QCD} m_b) \approx 17\ {\rm GeV}^2$. In fact, the fast increase of the form factors near the end-point region indicates that when q^2 becomes large or one gets too close to the physical states in the channel, light-cone expansion tends to break down.

- ⋆ Total decay width $(\text{in}\, 10^{-11} \times |V_{ub}|^2\, \text{GeV})$ for the semileptonic decays $\Lambda_b \to p\ell\bar{\nu}$ in the full QCD. Γ is obtained by integrating in the whole kinematical region using sum rule data, while Γ_f using the $q^2 < 16$GeV^2 fitted form factors extrapolated to the whole region. The error only reflects the variation of the Borel parameter and the continuum threshold between $6 < M_B^2 < 9$GeV^2 and $38 < s_0 < 40$GeV^2..

	asymptotic			QCD sum rule		
m_b(GeV)	4.7	4.8	4.9	4.7	4.8	4.9
Γ	1.5 ± 0.7	1.0 ± 0.4	0.7 ± 0.2	3.7 ± 2.2	3.1 ± 1.8	2.6 ± 1.4
Γ_f	0.56 ± 0.14	0.44 ± 0.09	0.35 ± 0.06	2.4 ± 1.3	2.0 ± 1.0	1.7 ± 0.8

- Numerical results for $\Lambda_c \to \Lambda \ell^+ \nu$
  - ⋆ In the whole kinematical region, $0 < q^2 < (M_{\Lambda_c} - M)^2$, both the form factors can be fitted well by the dipole formula

$$f_i(q^2) = \frac{f_i(0)}{a_2(q^2/M_{\Lambda_c}^2)^2 + a_1 q^2/M_{\Lambda_c}^2 + 1},$$

  - ⋆ The coefficients for two sets of parameters:

	twist-3			up to twist-6		
	a_2	a_1	$f_i(0)$	a_2	a_1	$f_i(0)$
f_1	1.595	-2.203	0.449	0.993	-1.712	0.392
f_2	2.992	-3.329	0.193	0.238	-1.339	-0.083

  - ⋆ Total decay width for the decay $\Lambda_c \to \Lambda \ell^+ \nu$: the twist-3 result is $\Gamma = (7.2 \pm 2.0) \times 10^{-14}$GeV while the up to twist-6 one is $\Gamma = (6.8 \pm 2.0) \times 10^{-14}$GeV. As to the asymmetry parameter, the twist-3 DAs correspond to $\alpha = -0.88 \pm 0.03$ and the up to twist-6 DAs give $\alpha = -(0.54 \pm 0.02)$.

Discussion and conclusion

- Light-cone QCD sum rule has been extended to study heavy-to-light transitions of baryons, the results are preliminary. More studies are needed and in particular radiative corrections to the sum rules have to be calculated.

- The accuracy of the results is about 20%, half of error comes from the parameters of the DAs, half the system error.

- A systematic study of baryon distribution amplitudes is mandatory.

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Thank You!