RGE Running of Leptonic CP-Violating Phases

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@

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Why RGEs ?

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High Energy

- Unified Theory
- Flavor Symmetries

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- Precise Neutrino Experiments

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provide a window to physics at very high energy scales

needed by precise tests of unified flavor models in the future

Content

- RGEs below the seesaw scale
- Radiative generation of three phases
- Quasi-fixed point of δ in the RGE running
- Summary

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S. Luo, J. W. Mei, Z. Z. Xing, **hep-ph/0507065** (PRD, 2005)
S. Luo, Z. Z. Xing, **hep-ph/0509065** (PLB, 2006)
S. Luo, Z. Z. Xing, **hep-ph/0603091** (PLB, 2006)

RGEs Below the Seesaw Scale

The effective Lagrangian:

$$-\mathcal{L} = \overline{E_L} H_1 Y_l l_R - \frac{1}{2}\overline{E_L} H_2 \cdot \kappa \cdot H_2^{c\dagger} E_L^c \ + \ \text{h.c.}$$

after SSB at EW scale: $M_l = v Y_l \cos\beta, \ M_\nu = v^2 \kappa \sin^2\beta$

(Here tanβ = <H_2> / < H_1> in the **MSSM**)

One-loop renormalization equation of κ

$$16\pi^2 \frac{d\kappa}{dt} = \alpha\kappa + \left(Y_l Y_l^\dagger\right)\kappa + \kappa\left(Y_l Y_l^\dagger\right)^T \qquad (\ t \equiv \ln(\mu/\Lambda_{SS}) \ )$$

with $\alpha = -\frac{6}{5} g_1^2 - 6 g_2^2 + 6\left(y_u^2 + y_c^2 + y_t^2\right)$, ($g_1$, g_2 denote the gauge couplings)

For more details: Babu *et al*, hep-ph/9309223, (PLB, 1993);
Chankowski *et al*, hep-ph/9306333, (PLB, 1993);
and for full scale, e.g., Antusch *et al*, hep-ph/0501272, (JHEP, 2005).

- In the flavor basis where Y_l is diagonal

$$\kappa = V \overline{\kappa} V^T \quad \text{with} \quad \overline{\kappa} = \text{Diag}\{\kappa_1, \kappa_2, \kappa_3\}$$

V is just the Maki-Nakagawa-Sakata matrix, and $m_i = v^2 \kappa_i \sin^2\beta$.

- The RGEs of κ_i (small contributions from y_μ, y_e are neglected)

$$\frac{d\kappa_1}{dt} = \frac{\kappa_1}{16\pi^2}\left[\alpha + 2y_\tau^2\left(s_{12}^2 s_{23}^2 - 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + c_{12}^2 c_{23}^2 s_{13}^2\right)\right]$$

$$\frac{d\kappa_2}{dt} = \frac{\kappa_2}{16\pi^2}\left[\alpha + 2y_\tau^2\left(c_{12}^2 s_{23}^2 + 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + c_{23}^2 s_{12}^2 s_{13}^2\right)\right]$$

$$\frac{d\kappa_3}{dt} = \frac{\kappa_3}{16\pi^2}\left[\alpha + 2y_\tau^2 c_{23}^2 c_{13}^2\right]$$

$y_\tau^2 \ll \alpha$

$y_\tau^2/(16\pi^2) = m_\tau^2(1+\tan^2\beta)/(16\pi^2 v^2) \approx 6.6 \times 10^{-7}(1+\tan^2\beta)$ in the MSSM

- Flavor-dependent RGE running effects are strongly suppressed.
- RGE running behaviors of three neutrino masses are almost identical.

Phase Conventions

- Standard Parametrization advocated by PDG

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \underline{s_{13}e^{-i\delta}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Parametrization advocated by our group

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \underline{s_{13}} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Phase relation between two conventions:

$\delta = \delta$, $\rho = \delta + \alpha_1 / 2$, $\rho = \delta + \alpha_2 / 2$,

three mixing angles in our parametrization equal to their counterparts in the Standard Parametrization.

Phase Conventions

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$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Parametrization advocated by our group

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Phase relation between two conventions:

$\delta = \delta$, $\rho = \delta + \alpha_1 / 2$, $\rho = \delta + \alpha_2 / 2$,

three mixing angles in our parametrization equal to their counterparts in the Standard Parametrization.

- <u>Similarities</u>

1. three mixing angles have simplest connection with neutrino oscillations (θ_{12} solar, θ_{23} atmospheric, θ_{13} reactor to the leading order).

2. the Jarlskog parameter has the identical expression.

$$\mathcal{J} = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 s_\delta \ , \qquad \text{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) = \mathcal{J} \sum_{\gamma,k} \left(\epsilon_{\alpha\beta\gamma} \epsilon_{ijk}\right)$$

- <u>Differences</u>

1. the effective mass of the $0\nu\beta\beta$ decay: $\langle m \rangle_{ee} = \left| m_1 V_{11}^2 + m_2 V_{12}^2 + m_3 V_{13}^2 \right|$

PDG: $\langle m \rangle_{ee} = \left| m_1 c_{12}^2 c_{13}^2 e^{i\alpha_1} + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_2} + m_3 s_{13}^2 e^{-2i\delta} \right|$ dependent on δ,

Ours: $\langle m \rangle_{ee} = \left| m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2 \right|$ independent of δ.

2. when $\theta_{13} \to 0$, δ automatically disappears in the PDG convention, good for discussing the quasi-fixed point of its RGE.

- One-loop RGEs of three mixing angles

$$\zeta_{ij} \equiv \frac{\kappa_i - \kappa_j}{\kappa_i - \kappa_j}$$

$$\begin{aligned}
\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2} \Bigg\{ & \frac{c_{(\rho-\sigma)}}{\zeta_{12}} \left[c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(c_{(\delta+\rho-\sigma)} c_{12}^2 - c_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \\
& + \zeta_{12} s_{(\rho-\sigma)} \left[s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(s_{(\delta+\rho-\sigma)} c_{12}^2 + s_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \\
& - \left[\frac{c_\rho}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) - \zeta_{13} s_\rho \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] c_{23} s_{12} s_{13} \\
& - \left[\frac{c_\sigma}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) - \zeta_{23} s_\sigma \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] c_{12} c_{23} s_{13} \Bigg\}
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}\theta_{23}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2} \Bigg\{ & \left[\frac{c_{(\delta-\rho)}}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) + \zeta_{13} s_{(\delta-\rho)} \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] s_{12} \\
& + \left[\frac{c_{(\delta-\sigma)}}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) + \zeta_{23} s_{(\delta-\sigma)} \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] c_{12} \Bigg\}
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}\theta_{13}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2} \Bigg\{ & - \left[\frac{c_\rho}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) - \zeta_{13} s_\rho \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] c_{12} c_{13} \\
& + \left[\frac{c_\sigma}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) - \zeta_{23} s_\sigma \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] c_{13} s_{12} \Bigg\}
\end{aligned}$$

Of three mixing angles, θ_{12} is most sensitive to RGE effects.

• One-loop RGEs of three mixing angles

$$\zeta_{ij} \equiv \frac{\kappa_i - \kappa_j}{\kappa_i - \kappa_j}$$

$$\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\left\{\frac{c_{(\rho-\sigma)}}{\zeta_{12}}\left[c_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2 - \right.\right.\right.$$
$$+\zeta_{12}s_{(\rho-\sigma)}\left[s_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2 - c_{\cdots}\right.\right.$$
$$-\left[\frac{c_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23} - c_\rho c_{12}c_{23}s_{13}\right.\right.$$
$$-\left[\frac{c_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23} + c_\sigma c_{23}s_{12}s_{1\cdots}\right.\right.$$

$$\frac{\mathrm{d}\theta_{23}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2}\left\{\left[\frac{c_{(\delta-\rho)}}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23} - c_\rho c_{\cdots}\right.\right.\right.$$
$$+\left[\frac{c_{(\delta-\sigma)}}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23} + c_\sigma c_{23}s_{1\cdots}\right.\right.$$

$$\frac{\mathrm{d}\theta_{13}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2}\left\{-\left[\frac{c_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23} - c_\rho c_{\cdots}\right.\right.\right.$$
$$+\left[\frac{c_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23} + c_\sigma c_{23}s_{12}s_{1\cdots}\right.\right.$$

- **Hierarchy**
  (e.g., $m_1 \ll m_2 \ll m_3$ and $m_1 = 10^{-3}$ eV)
  $\zeta_{12}^{-1} \sim -1.25, \quad \zeta_{13}^{-1} \sim -1.0, \quad \zeta_{23}^{-1} \sim -1.5$

- **Degenerate**
  (e.g., $m_1 \lesssim m_2 \lesssim m_3$ and $m_1 = 0.2$ eV)
  $\zeta_{12}^{-1} \sim -2001, \quad \zeta_{13}^{-1} \sim -64, \quad \zeta_{23}^{-1} \sim -66$

Of three mixing angles, θ_{12} is most sensitive to RGE effects.

- # One-loop RGEs of three mixing angles

$$\zeta_{ij} \equiv \frac{\kappa_i - \kappa_j}{\kappa_i - \kappa_j}$$

$$\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\left\{\frac{c_{(\rho-\sigma)}}{\zeta_{12}}\left[c_{(\rho-\sigma)} c_{12} s_{12}\left(s_{23}^2 - \cdots\right.\right.\right.$$
$$+\zeta_{12} s_{(\rho-\sigma)}\left[s_{(\rho-\sigma)} c_{12} s_{12}\left(s_{23}^2 - c_{2}^{2} \cdots\right.\right.$$
$$-\left[\frac{c_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \cdots\right.\right.$$
$$-\left[\frac{c_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{1} \cdots\right.\right.$$

$$\frac{\mathrm{d}\theta_{23}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2}\left\{\left[\frac{c_{(\delta-\rho)}}{\zeta_{13}}\left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c \cdots\right.\right.\right.$$
$$+\left[\frac{c_{(\delta-\sigma)}}{\zeta_{23}}\left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{1} \cdots\right.\right.$$

$$\frac{\mathrm{d}\theta_{13}}{\mathrm{d}t} = \frac{y_\tau^2 c_{23}}{16\pi^2}\left\{-\left[\frac{c_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c \cdots\right.\right.\right.$$
$$+\left[\frac{c_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{1} \cdots\right.\right.$$

- **Hierarchy**
  (e.g., $m_1 \ll m_2 \ll m_3$ and $m_1 = 10^{-3}$ eV)
  $\zeta_{12}^{-1} \sim -1.25, \quad \zeta_{13}^{-1} \sim -1.0, \quad \zeta_{23}^{-1} \sim -1.5$
- **Degenerate**
  (e.g., $m_1 \lesssim m_2 \lesssim m_3$ and $m_1 = 0.2$ eV)
  $\zeta_{12}^{-1} \sim -2001, \quad \zeta_{13}^{-1} \sim -64, \quad \zeta_{23}^{-1} \sim -66$

- RGE evolution has strong effects on neutrino mixing parameters only in the case that three neutrino masses are nearly degenerate.

Of three mixing angles, θ_{12} is most sensitive to RGE effects.

- One-loop RGE of δ

$$
\begin{aligned}
\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\Bigg\{ & \frac{s_{(\rho-\sigma)}}{\zeta_{12}}\left[c_{(\rho-\sigma)}\left(s_{23}^2 - c_{23}^2 s_{13}^2\right) - \left(c_{(\delta+\rho-\sigma)}c_{12}^2 - c_{(\delta-\rho+\sigma)}s_{12}^2\right)\frac{c_{23}s_{23}s_{13}}{c_{12}s_{12}}\right] \\
& -\zeta_{12}c_{(\rho-\sigma)}\left[s_{(\rho-\sigma)}\left(s_{23}^2 - c_{23}^2 s_{13}^2\right) - \left(s_{(\delta+\rho-\sigma)}c_{12}^2 + s_{(\delta-\rho+\sigma)}s_{12}^2\right)\frac{c_{23}s_{23}s_{13}}{c_{12}s_{12}}\right] \\
& +\frac{1}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23} - c_\rho c_{12}c_{23}s_{13}\right)\left[\frac{s_\rho c_{23}}{c_{12}s_{13}}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right) - s_{(\delta-\rho)}\frac{s_{12}}{s_{23}}\left(c_{23}^2 - s_{23}^2\right)\right] \\
& +\zeta_{13}\left(s_{(\delta-\rho)}s_{12}s_{23} + s_\rho c_{12}c_{23}s_{13}\right)\left[\frac{c_\rho c_{23}}{c_{12}s_{13}}\left(c_{12}^2 - s_{12}^2 s_{13}^2\right) + c_{(\delta-\rho)}\frac{s_{12}}{s_{23}}\left(c_{23}^2 - s_{23}^2\right)\right] \\
& -\frac{1}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23} + c_\sigma c_{23}s_{12}s_{13}\right)\left[\frac{s_\sigma c_{23}}{s_{12}s_{13}}\left(s_{12}^2 - c_{12}^2 s_{13}^2\right) + s_{(\delta-\sigma)}\frac{c_{12}}{s_{23}}\left(c_{23}^2 - s_{23}^2\right)\right] \\
& -\zeta_{23}\left(s_{(\delta-\sigma)}c_{12}s_{23} - s_\sigma c_{23}s_{12}s_{13}\right)\left[\frac{c_\sigma c_{23}}{s_{12}s_{13}}\left(s_{12}^2 - c_{12}^2 s_{13}^2\right) - c_{(\delta-\sigma)}\frac{c_{12}}{s_{23}}\left(c_{23}^2 - s_{23}^2\right)\right]\Bigg\}
\end{aligned}
$$

• The RGE evolution of δ depends on ρ and σ while the RGE evolution of ρ and σ depends on δ.
• Three CP-violating phases entangled with one another in the one-loop RGE evolution. That's why radiactive generation of δ, ρ, σ is in general possible.

- One-loop RGEs of ρ, σ

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\left\{\frac{s_{(\rho-\sigma)}}{\zeta_{12}}\left[c_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2-c_{23}^2s_{13}^2\right)-\left(c_{(\delta+\rho-\sigma)}c_{12}^2-c_{(\delta-\rho+\sigma)}s_{12}^2\right)c_{23}s_{23}s_{13}\right]\frac{s_{12}}{c_{12}}\right.$$
$$-\zeta_{12}c_{(\rho-\sigma)}\left[s_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2-c_{23}^2s_{13}^2\right)-\left(s_{(\delta+\rho-\sigma)}c_{12}^2+s_{(\delta-\rho+\sigma)}s_{12}^2\right)c_{23}s_{23}s_{13}\right]\frac{s_{12}}{c_{12}}$$
$$+\left[\frac{s_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23}-c_\rho c_{12}c_{23}s_{13}\right)+\zeta_{13}c_\rho\left(s_{(\delta-\rho)}s_{12}s_{23}+s_\rho c_{12}c_{23}s_{13}\right)\right]\frac{c_{23}\left(c_{12}^2c_{13}^2-s_{13}^2\right)}{c_{12}s_{13}}$$
$$\left.-\left[\frac{s_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23}+c_\sigma c_{23}s_{12}s_{13}\right)+\zeta_{23}c_\sigma\left(s_{(\delta-\sigma)}c_{12}s_{23}-s_\sigma c_{23}s_{12}s_{13}\right)\right]\frac{c_{23}c_{13}^2s_{12}}{s_{13}}\right\},$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\left\{\frac{s_{(\rho-\sigma)}}{\zeta_{12}}\left[c_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2-c_{23}^2s_{13}^2\right)-\left(c_{(\delta+\rho-\sigma)}c_{12}^2-c_{(\delta-\rho+\sigma)}s_{12}^2\right)c_{23}s_{23}s_{13}\right]\frac{c_{12}}{s_{12}}\right.$$
$$-\zeta_{12}c_{(\rho-\sigma)}\left[s_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2-c_{23}^2s_{13}^2\right)-\left(s_{(\delta+\rho-\sigma)}c_{12}^2+s_{(\delta-\rho+\sigma)}s_{12}^2\right)c_{23}s_{23}s_{13}\right]\frac{c_{12}}{s_{12}}$$
$$-\left[\frac{s_\sigma}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23}+c_\sigma c_{23}s_{12}s_{13}\right)+\zeta_{23}c_\sigma\left(s_{(\delta-\sigma)}c_{12}s_{23}-s_\sigma c_{23}s_{12}s_{13}\right)\right]\frac{c_{23}\left(c_{13}^2s_{12}^2-s_{13}^2\right)}{s_{12}s_{13}}$$
$$\left.+\left[\frac{s_\rho}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23}-c_\rho c_{12}c_{23}s_{13}\right)+\zeta_{13}c_\rho\left(s_{(\delta-\rho)}s_{12}s_{23}+s_\rho c_{12}c_{23}s_{13}\right)\right]\frac{c_{12}c_{23}c_{13}^2}{s_{13}}\right\}$$

- One-loop RGE of $\mathcal{J}$ (depends on δ, ρ and σ)

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{J}}{\mathrm{d}t} = \frac{y_\tau^2}{16\pi^2}\Big\{ & -\frac{c_{23}c_{13}^2 s_{23}s_{13}}{\zeta_{12}}\left[c_\delta s_{(\rho-\sigma)} + c_{(\rho-\sigma)}s_\delta\left(c_{12}^2 - s_{12}^2\right)\right]\left[c_{(\delta+\rho-\sigma)}c_{12}^2 c_{23}s_{23}s_{13}\right. \\
& \left. -c_{(\delta-\rho+\sigma)}c_{23}s_{12}^2 s_{23}s_{13} - c_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2 - c_{23}^2 s_{13}^2\right)\right] \\
& +\zeta_{12}c_{23}c_{13}^2 s_{23}s_{13}\left[c_\delta c_{(\rho-\sigma)} - s_\delta s_{(\rho-\sigma)}\left(c_{12}^2 - s_{12}^2\right)\right]\left[s_{(\delta+\rho-\sigma)}c_{12}^2 c_{23}s_{23}s_{13}\right. \\
& \left. +s_{(\delta-\rho+\sigma)}c_{23}s_{12}^2 s_{23}s_{13} - s_{(\rho-\sigma)}c_{12}s_{12}\left(s_{23}^2 - c_{23}^2 s_{13}^2\right)\right] \\
& +\frac{c_{23}c_{13}^2 s_{12}}{\zeta_{13}}\left(c_{(\delta-\rho)}s_{12}s_{23} - c_\rho c_{12}c_{23}s_{13}\right)\left[s_\rho c_{12}s_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right)\right. \\
& \left. +s_{(\delta-\rho)}c_{23}s_{12}^2 s_{23}s_{13}^2 + \left[c_\delta s_\rho - c_\rho s_\delta\left(c_{13}^2 - s_{13}^2\right)\right]c_{12}^2 c_{23}s_{23}\right] \\
& +\zeta_{13}c_{23}c_{13}^2 s_{12}\left(s_{(\delta-\rho)}s_{12}s_{23} + s_\rho c_{12}c_{23}s_{13}\right)\left[c_\rho c_{12}s_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right)\right. \\
& \left. -c_{(\delta-\rho)}c_{23}s_{12}^2 s_{23}s_{13}^2 + \left[c_\delta c_\rho + s_\delta s_\rho\left(c_{13}^2 - s_{13}^2\right)\right]c_{12}^2 c_{23}s_{23}\right] \\
& -\frac{c_{12}c_{23}c_{13}^2}{\zeta_{23}}\left(c_{(\delta-\sigma)}c_{12}s_{23} + c_\sigma c_{23}s_{12}s_{13}\right)\left[\left(c_\delta s_\sigma - c_\sigma s_\delta c_{13}^2\right)c_{23}s_{12}^2 s_{23}\right. \\
& \left. -s_\sigma c_{12}s_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right) + \left(s_{(\delta-\sigma)}c_{12}^2 + c_\sigma s_\delta s_{12}^2\right)c_{23}s_{23}s_{13}^2\right] \\
& +\zeta_{23}c_{12}c_{23}c_{13}^2\left(s_{(\delta-\sigma)}c_{12}s_{23} - s_\sigma c_{23}s_{12}s_{13}\right)\left[-\left(c_\delta c_\sigma + s_\delta s_\sigma c_{13}^2\right)c_{23}s_{12}^2 s_{23}\right. \\
& \left. +c_\sigma c_{12}s_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right) + \left(c_{(\delta-\sigma)}c_{12}^2 + s_\delta s_\sigma s_{12}^2\right)c_{23}s_{23}s_{13}^2\right]\Big\}
\end{aligned}
$$

Numerical Calculations

- How do we do the numerical calculation?

  We follow a "running and diagonalizing" procedure: first compute the RGE evolution of lepton mass matrices and then extract their mass eigenvalues and flavor mixing parameters at $\Lambda_{\rm EW}$.

- Present information on neutrino masses and mixing from oscillation data (90% CL):

$\Delta m^2_{12} = (7.2 \sim 8.9) \times 10^{-5} eV^2$ central value: $8.0 \times 10^{-5} eV^2$

$\Delta m^2_{23} = (1.7 \sim 3.3) \times 10^{-3} eV^2$ central value: $2.5 \times 10^{-3} eV^2$

$$30^\circ < \theta_{12} < 38^\circ, \quad 36^\circ < \theta_{23} < 54^\circ, \quad \theta_{13} < 10^\circ$$

The engenvalues of Y_l and the elements of κ at Λ_{SS} are chosen in such a way that they can correctly run to their low energy values.

Radiative Generation of Phases

- We concentrate on the case that three neutrino masses are nearly degenerate and $\tan\beta = 10$.
- Approximate RGEs of three phases in this case

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} \approx \frac{y_\tau^2}{16\pi^2}\left[\frac{c_{(\rho-\sigma)}s_{(\rho-\sigma)}}{\zeta_{12}}s_{23}^2 + \left(\frac{c_{(\delta-\rho)}s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)}s_\sigma}{\zeta_{23}}\right)\frac{c_{12}s_{12}c_{23}s_{23}}{s_{13}}\right]$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} \approx \frac{y_\tau^2}{16\pi^2}\left[\frac{c_{(\rho-\sigma)}s_{(\rho-\sigma)}}{\zeta_{12}}s_{12}^2 s_{23}^2 + \left(\frac{c_{(\delta-\rho)}s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)}s_\sigma}{\zeta_{23}}\right)\frac{c_{12}s_{12}c_{23}s_{23}}{s_{13}}\right]$$

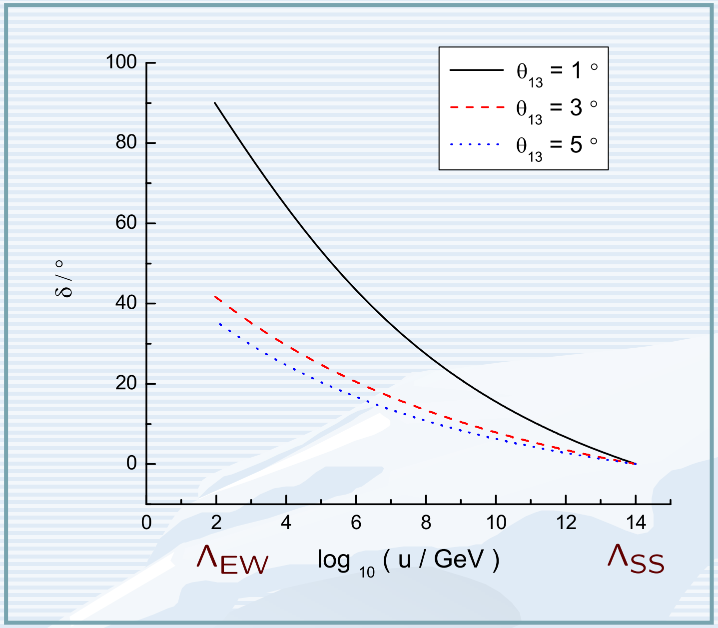
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} \approx \frac{y_\tau^2}{16\pi^2}\left[\frac{c_{(\rho-\sigma)}s_{(\rho-\sigma)}}{\zeta_{12}}c_{12}^2 s_{23}^2 + \left(\frac{c_{(\delta-\rho)}s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)}s_\sigma}{\zeta_{23}}\right)\frac{c_{12}s_{12}c_{23}s_{23}}{s_{13}}\right]$$

The one-loop RGE running behaviors of three CP-violating phases are quite similar in the chosen parametrization.

• Radiative Generation of δ = 90°

• δ = 90° might imply "Maximal" CP violation in some sense.

• In the inverted hierarchy case, radiative generation of δ (or ρ, σ) = 90° from 0° is also possible.

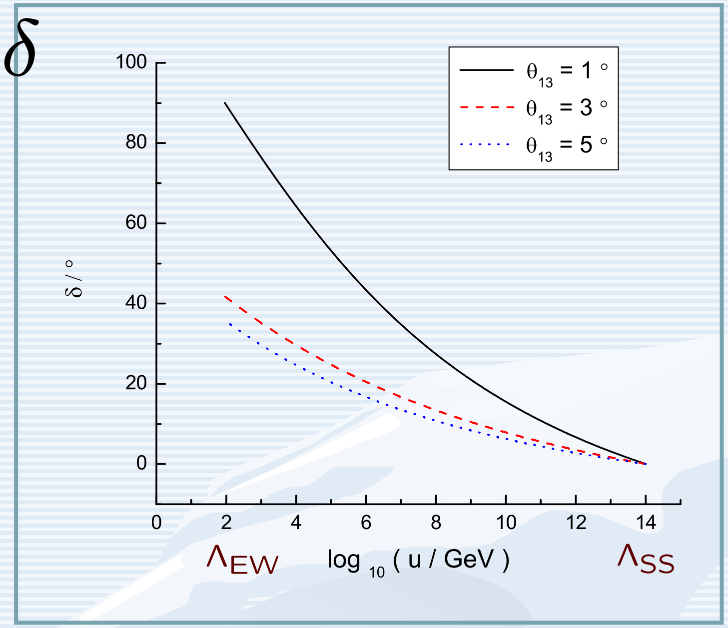


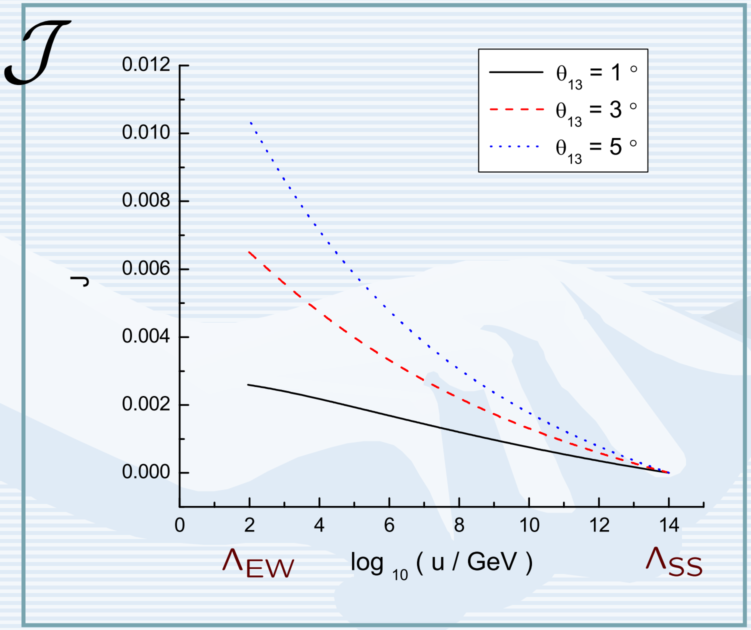
Parameter	**Seesaw Scale** (Input)	**Electroweak Scale** (Output)		
		$\theta_{13} = 1^\circ$	$\theta_{13} = 3^\circ$	$\theta_{13} = 5^\circ$
m_1(eV)	0.241	0.20	0.20	0.20
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	20.4	7.79	7.17	6.56
$\Delta m^2_{31}(10^{-3}\ \mathrm{eV}^2)$	3.32	2.20	2.20	2.20
θ_{12}	24.1°	33.0°	33.0°	33.1°
θ_{23}	43.9°	45.1°	45.0°	45.0°
θ_{13}	1°/3°/5°	0.65°	2.46°	4.52°
δ	0°	90.0°	41.8°	35.8°
ρ	4.0°	72.2°	23.8°	17.6°
σ	−57.5°	26.3°	−22.0°	−28.1°

• Radiative Generation of $\delta = 90°$

• $\delta = 90°$ might imply "Maximal" CP violation in some sense.

• In the inverted hierarchy case, radiative generation of δ (or ρ, σ) = $90°$ from $0°$ is also possible.





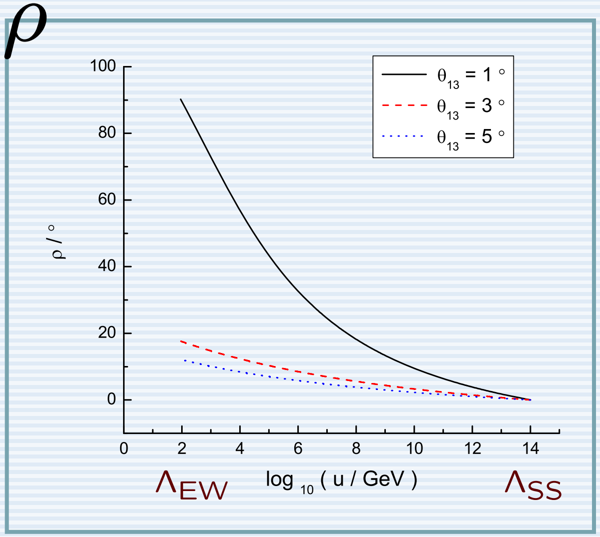
• $\mathcal{J}$ is sensitive to both δ and θ_{13}.

• $\mathcal{J}$ does not diverge when $\theta_{13} \to 0$.

• With a larger value of θ_{13}, δ runs faster but $\mathcal{J}$ slower.

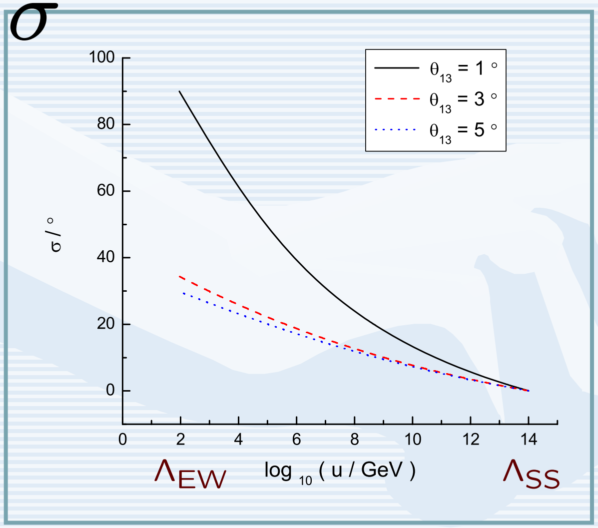
• Radiative Generation of $\rho = 90^\circ$, $\sigma = 90^\circ$

ρ



Parameter	Input (Λ_{SS})	Output (Λ_{EW})		
		$\theta_{13} = 1^\circ$	$\theta_{13} = 3^\circ$	$\theta_{13} = 5^\circ$
m_1(eV)	0.241	0.20	0.20	0.20
$\Delta m^2_{21}(10^{-5}\ \text{eV}^2)$	20.4	8.54	7.90	7.27
$\Delta m^2_{31}(10^{-3}\ \text{eV}^2)$	3.32	2.21	2.20	2.20
θ_{12}	27.6°	33.1°	33.2°	33.3°
θ_{23}	43.9°	44.8°	44.8°	44.8°
θ_{13}	1°/3°/5°	0.43°	2.17°	4.24°
δ	0°	107.6°	35.4°	30.2°
ρ	0°	90.2°	17.6°	12.1°
σ	−67.7°	34.1°	−38.3°	−44.6°

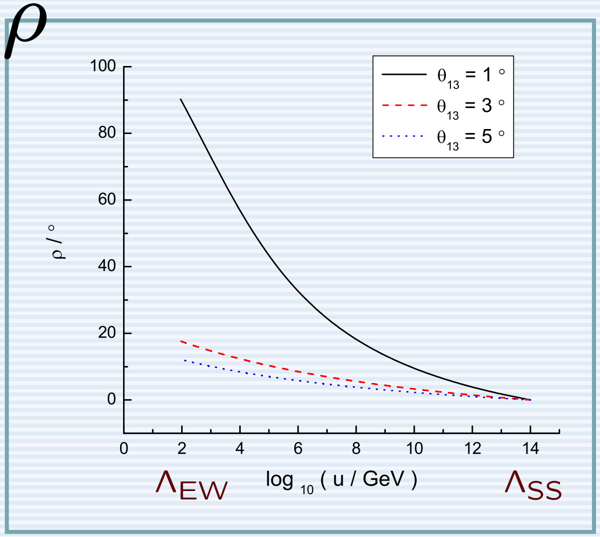
σ



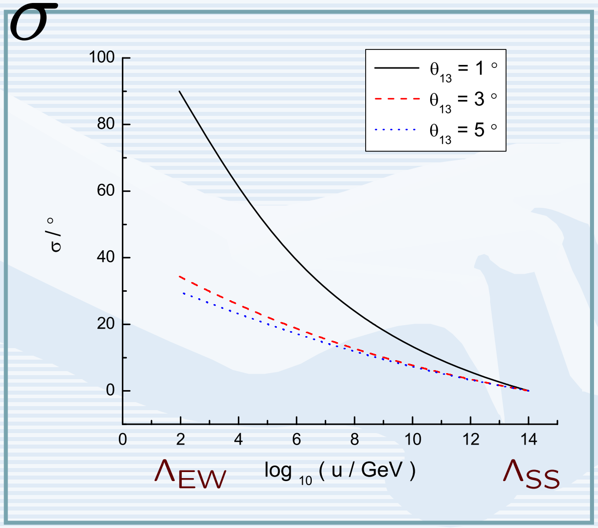
Parameter	Input (Λ_{SS})	Output (Λ_{EW})		
		$\theta_{13} = 1^\circ$	$\theta_{13} = 3^\circ$	$\theta_{13} = 5^\circ$
m_1(eV)	0.241	0.20	0.20	0.20
$\Delta m^2_{21}(10^{-5}\ \text{eV}^2)$	20.3	8.24	8.85	9.50
$\Delta m^2_{31}(10^{-3}\ \text{eV}^2)$	3.32	2.21	2.21	2.21
θ_{12}	24.1°	33.2°	34.2°	35.2°
θ_{23}	43.9°	44.6°	44.6°	44.6°
θ_{13}	1°/3°/5°	0.51°	2.27°	4.29°
δ	119.7°	216.0°	161.0°	157.1°
ρ	60.8°	135.3°	78.8°	73.6°
σ	0°	90.0°	34.4°	29.8°

- Radiative Generation of $\rho = 90^\circ$, $\sigma = 90^\circ$

ρ



σ



Remarks:

Simultaneous generation of appreciable

- δ and ρ from $\sigma \neq 0^\circ$,
  is **possible**;
- δ and σ from $\rho \neq 0^\circ$,
  is **possible**;
- ρ and σ from $\delta \neq 0^\circ$,
  is **suppressed**.

Quisi-fixed Point of δ

- $\theta_{13} \to 0$ may naturally arise from an underlying flavor symmetry, and is allowed by present experimental data. (deserve careful considerations)
- Note that the RGE of δ contains terms of s_{13}^{-1}. —— **divergence**
- But the derivative of δ can keep finite in the limit $\theta_{13} \to 0$, if three CPV phases satisfy a novel continuity condition. ⟶ **quasi-fixed point of δ**
- The exact one-loop RGE of δ (included terms of y_μ and y_e) (PDG)

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{C\left(y_\tau^2 - y_\mu^2\right)}{32\pi^2} \cdot \frac{m_3 \chi}{\Delta m_{31}^2 \Delta m_{32}^2} \cdot \frac{\sin 2\theta_{12} \sin 2\theta_{23}}{\sin\theta_{13}} + \text{other terms}$$

and $\chi = m_3 \Delta m_{21}^2 \sin\delta + m_2 \Delta m_{31}^2 \sin(\delta + \alpha_2) - m_1 \Delta m_{32}^2 \sin(\delta + \alpha_1)$

- $\theta_{13} \to 0$ and keep $\mathrm{d}\delta/\mathrm{d}t$ finite

$\zeta \equiv \Delta m_{21}^2 / \Delta m_{32}^2$

1. $m_3 = 0$, —— δ has no quasi-fixed point and only $(\alpha_1 - \alpha_2)$ is physical.
2. $\chi = 0$, —— continuity condition: $\cot\delta = \frac{m_1 \cos\alpha_1 - (1+\zeta) m_2 \cos\alpha_2 - \zeta m_3}{(1+\zeta) m_2 \sin\alpha_2 - m_1 \sin\alpha_1}$

1) Normal hierachy $m_1 \ll m_2 \ll m_3$, and $m_1 = 0$

$$m_2 \sin\delta + m_3 \sin\left(\delta + \alpha_2\right) \approx 0$$

2) Inverted hierarchy $m_3 \ll m_1 \lesssim m_2$, and $m_3 \sim 0$

$$m_1 \sin\left(\delta + \alpha_2\right) \approx m_2 \sin\left(\delta + \alpha_1\right)$$

3) Near degeneracy ($\Delta m_{32}^2 > 0$ or $\Delta m_{32}^2 < 0$)

$$\sin\left(\delta + \alpha_1\right) \approx \sin\left(\delta + \alpha_2\right) \qquad (\alpha_1 \neq \alpha_2)$$

$$\Rightarrow \quad \delta \approx -(\alpha_1+\alpha_2)/2+(n+1/2)\pi \quad (n = 0, \pm 1, \pm 2, \cdots)$$

$$\sin\delta + \sin\left(\delta + \alpha_2\right) \approx 0 \qquad (\alpha_1 = \alpha_2)$$

$$\Rightarrow \quad \delta \approx -\alpha_2/2 + n\pi \qquad (n = 0, \pm 1, \pm 2, \cdots)$$

- Once the initial value of α_1 and α_2 are fixed, the value of δ at its quasi-fixed point can be determined.

Take Tri-bimaximal Mixing as an Example

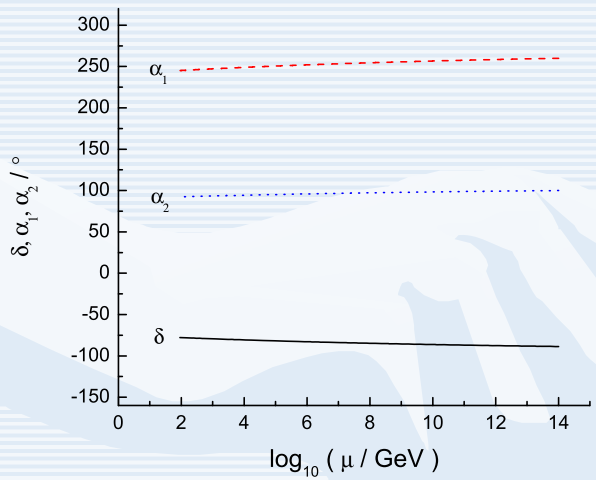
- Generalized tri-bimaximal neutrino mixing

$$V'_0 = \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

at Λ_{SS}, $\theta_{12} = 35.26^\circ$
$\theta_{23} = 45^\circ$
$\theta_{13} = 0^\circ$

- Quasi-fixed point of δ:

$$\hat{\delta} \approx -\frac{1}{2}\left(\hat{\alpha}_1 + \hat{\alpha}_2\right) + \left(n + \frac{1}{2}\right)\pi \,, \quad (\hat{\alpha}_1 \neq \hat{\alpha}_2) \quad \text{or} \quad \hat{\delta} \approx -\frac{\hat{\alpha}_1}{2} + n\pi \,, \quad (\hat{\alpha}_1 \approx \hat{\alpha}_2)$$



Parameter	Input at Λ_{SS}	Output at Λ_{EW}
m_1(eV)	0.241	0.201
$\Delta m^2_{21}(10^{-5}$ eV$^2)$	17.0	8.19
$\Delta m^2_{31}(10^{-3}$ eV$^2)$	3.3	2.21
θ_{12}	35.26°	36.38°
θ_{23}	45.0°	46.22°
θ_{13}	0°	1.367°
δ	—	−77.85°
α_1	260.0°	245.17°
α_2	100.0°	92.27°

Summary

- RGE evolution has appreciable effects on neutrino masses and mixing parameters, especially on three CP-violating phases and especially when three neutrino masses are nearly degenerate.
- Since three CP-violating phases entangled with one another in the one-loop RGE evolution, the radiative generation of one (or two) CP-violating phase(s) from the other is possible, even the maximal value(90°) is achievable in some case.
- The quasi-fixed point in the RGE running of δ is in general unavoidable for those neutrino mixing pattern with $\theta_{13} = 0^\circ$, hence it should be taken into account for model building at high scale.

Thanks !