Re-examining charmless $B \to PV$ decays in QCD factorization approach

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Outline

1. Introduction and Motivation
2. Decay amplitudes of $B \to PV$ at NLO in α_s
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4. Numerical results and Discussions
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Based on our recent works: Xinqiang Li and Yadong Yang, PRD73,114027(2006), PRD72,074007(2005).

1 Introduction and Motivations

The study of charmless hadronic B decays are of particular importance in current particle physics:

⋆ to understand the origin of CP violation;

⋆ to determine the flavor mixing parameters of the quark sector within the SM;

⋆ to probe possible new physics scenarios beyond the SM.

Experimentally, huge amount of experimental data has been analyzed with appreciative precision: BABAR and Belle experiments.

Theoretically, much progress has also been made: the perturbative QCD method, the QCD factorization (or BBNS approach); the soft collinear effective theory, and model-independent methods based on (approximate) flavor symmetries,....

At the present stage, some puzzles in rare hadronic B decays still exist:

- the large measured branching ratio of $B^0 \to \pi^0\pi^0$.
- the observed unmatched CP asymmetries: $|A_{\rm CP}(B^0 \to \pi^{\pm}K^{\mp}| \gg |A_{\rm CP}(B^{\pm} \to \pi^0 K^{\pm}|$.
- the abnormally large measured branching ratios of $B \to \eta' K$ and $B \to \eta K^*$ decays;
- the large transverse polarization fractions in $B \to \phi K^*$ decays;
- ...

Except the first one, all these other decay modes are dominated by the FCNC $b \to s$ transitions, which are sensitive to NP effects. Confronted with these anomalies, theorists are forced not only to consider more precise QCD effects, but also to speculate on possible NP effects.

Here, we consider contributions of the higher order penguin contractions of spectator-scattering amplitudes induced by the $b \to s(d)g^*g^*$ transitions.

- ⋆ Contributions of the $b \to sgg$ process to the inclusive and semi-inclusive B decays could be large compared to $b \to sg$ process: W. S. Hou, NPB308, 561 (1988); H. Simma and D. Wyler, NPB(1990); C. Greub and P. Liniger, PRD63, 054025 (2001); G. Eilam and Y. D. Yang, PRD66, 074010 (2002).;
- ⋆ These higher order penguin contraction contributions are not negligible in exclusive hadronic B decays, especially in penguin-dominated decay modes: Xinqiang Li and Yadong Yang, PRD72,074007(2005);
- ⋆ A lot of new Quasi two-body $B \to PV$ decays have been measured experimentally.

To further investigate their impacts on exclusive hadronic B decays, we consider charmless $B \to PV$ decays (P: pseudoscalar meson, V: vector meson).

Because of their similar flavor structures, $B \to PV$ decays are closely related to their PP counterparts. However, these modes have apparent advantages in some cases:

- ⋆ due to the less penguin pollution, $B \to \pi\rho$ decay modes are more suitable than $B \to \pi\pi$ ones for extracting the weak angle α of the CKM UT.
- ⋆ the penguin contributions in PV decay modes are generally smaller than in their PP counterparts, thus studies of the penguin-dominated PV decay modes may be helpful to discuss sub-leading amplitude contributions, such as the annihilation contributions.

2 Decay amplitudes at NLO in α_s

Using the OPE and the RGE, the low energy effective for charmless B decays are

$$\begin{aligned}\mathcal{H}_{\rm eff} &= \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{uq}^*\left(C_1O_1^u + C_2O_2^u\right) + V_{cb}V_{cq}^*\left(C_1O_1^c + C_2O_2^c\right) \\ &\quad - V_{tb}V_{tq}^*\Big(\sum_{i=3}^{10} C_iO_i + C_{7\gamma}O_{7\gamma} + C_{8g}O_{8g}\Big)\Big] + \text{h.c.}, \end{aligned} \tag{1}$$

where the Wilson coefficients $C_i(\mu)$ represent all the physics contributions higher than the scale $\mu \sim \mathcal{O}(m_b)$. The effective operators Q_i govern a given decay process, including current-current operators, QCD (EW)-penguin operators, and dipole operators.

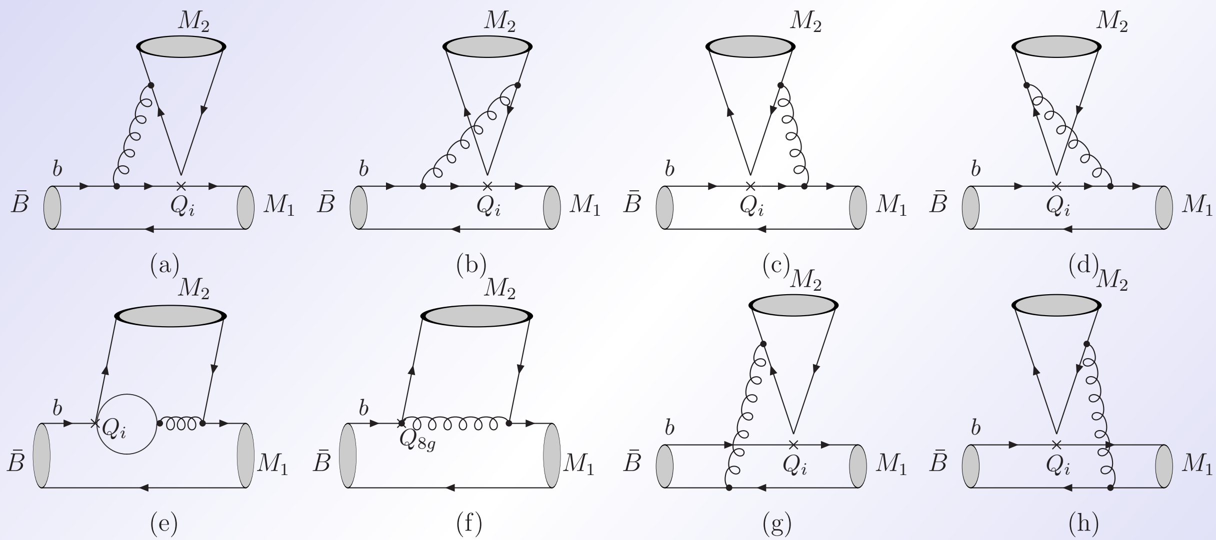
With the effective Hamiltonian Eq. (1), the decay amplitude for a general $B \to PV$ decay can be written as

$$\langle PV|\mathcal{H}_{\rm eff}|B\rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(\prime)}\, C_i \,\langle PV|Q_i^p|B\rangle \,. \tag{2}$$

With the QCDF approach, the hadronic matrix elements can be factorized

$$\begin{aligned}\langle PV|Q_i^p|B\rangle \;&=\; F_+^{B\to P}\, T_{V,i}^{\rm I} * f_V\, \Phi_V + A_0^{B\to V}\, T_{P,i}^{\rm I} * f_P\, \Phi_P \\ &\quad + T_i^{\rm II} * f_B\, \Phi_B * f_P\, \Phi_P * f_V\, \Phi_V \,,\end{aligned} \tag{3}$$

where Φ_M are the meson LCDAs, $F_+^{B\to P}$ and $A_0^{B\to V}$ are $B \to P$ and $B \to V$ transition form factors. The kernels $T_i^{\rm I,II}$ are dominated by hard gluon exchanges, and hence calculable perturbatively. The relevant Feynman diagrams are shown in the next slide.



Order α_s corrections to the hard-scattering kernels $T^{\mathrm{I}}_{M,i}$ (coming from the diagrams (a)-(f)) and T^{II}_i (coming from the last two diagrams).

After direct calculations, the decay amplitude for a general $B \to PV$ decay can then be rewritten as

$$\mathcal{A}(B \to PV) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1}^{10} \lambda_p^{(\prime)}\, a_i^p \, \langle PV|Q_i|B\rangle_F \,, \qquad (4)$$

All the "nonfactorizable" effects are encoded in the coefficients a_i^p:

$$\begin{aligned} a_i^p(M_1M_2) \;&=\; (C_i + \frac{C_{i\pm1}}{N_c})N_i(M_2) \\ &\quad + \frac{C_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right] + P_i^p(M_2)\,. \end{aligned} \qquad (5)$$

The quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1M_2)$ for hard-spectator interactions, and $P_i^p(M_2)$ for penguin contributions.

The coefficients a_i^p $(i = 3, ..., 10)$ always appear in pairs. So, one can define the following quantities α_i^p in terms of a_i^p:

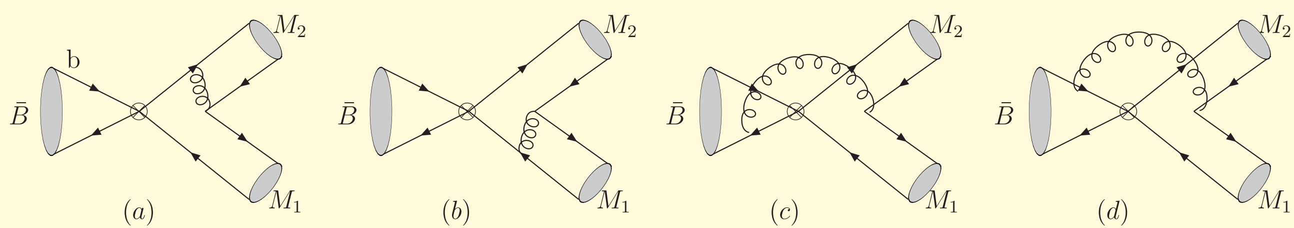
$$
\begin{aligned}
\alpha_1(M_1M_2) &= a_1(M_1M_2)\,, \quad \alpha_2(M_1M_2) = a_2(M_1M_2)\,, \\
\alpha_3^p(M_1M_2) &= \begin{cases} a_3^p(M_1M_2) - a_5^p(M_1M_2)\,; & \text{if } M_1M_2 = VP\,, \\ a_3^p(M_1M_2) + a_5^p(M_1M_2)\,; & \text{if } M_1M_2 = PV\,, \end{cases} \\
\alpha_4^p(M_1M_2) &= \begin{cases} a_4^p(M_1M_2) + r_\chi^{M_2}\, a_6^p(M_1M_2)\,; & \text{if } M_1M_2 = PV\,, \\ a_4^p(M_1M_2) - r_\chi^{M_2}\, a_6^p(M_1M_2)\,; & \text{if } M_1M_2 = VP\,, \end{cases} \\
\alpha_{3,ew}^p(M_1M_2) &= \begin{cases} a_9^p(M_1M_2) - a_7^p(M_1M_2)\,; & \text{if } M_1M_2 = VP\,, \\ a_9^p(M_1M_2) + a_7^p(M_1M_2)\,; & \text{if } M_1M_2 = PV\,, \end{cases} \\
\alpha_{4,ew}^p(M_1M_2) &= \begin{cases} a_{10}^p(M_1M_2) + r_\chi^{M_2}\, a_8^p(M_1M_2)\,; & \text{if } M_1M_2 = PV\,, \\ a_{10}^p(M_1M_2) - r_\chi^{M_2}\, a_8^p(M_1M_2)\,; & \text{if } M_1M_2 = VP\,, \end{cases}
\end{aligned}
\tag{6}
$$

It should be noted that for different final states, the vector and the scalar penguin amplitudes have different interference effects.

The annihilation contributions are power suppressed compared to the leading order terms, and hence do not appear in Eq. (3). Nevertheless, these contributions may be numerically important for realistic B-meson decays. So, we also take into account their contributions.

$$\mathcal{A}^{ann}(B \to PV) \propto \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i} \lambda_p^{(\prime)} f_B\, f_{M_1}\, f_{M_2}\, b_i(M_1 M_2)\,, \tag{7}$$

At order $\mathcal{O}(\alpha_s)$, the annihilation kernels $b_i(M_1 M_2)$ arise from the following four diagrams:

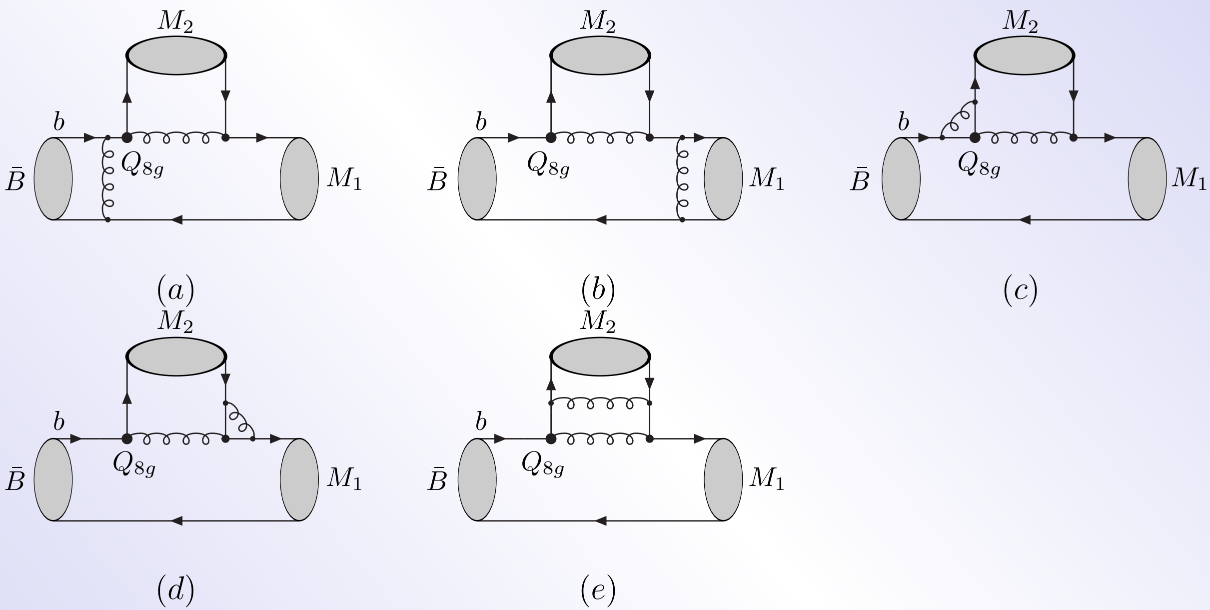


Comments on the NLO results within the QCDF framework:

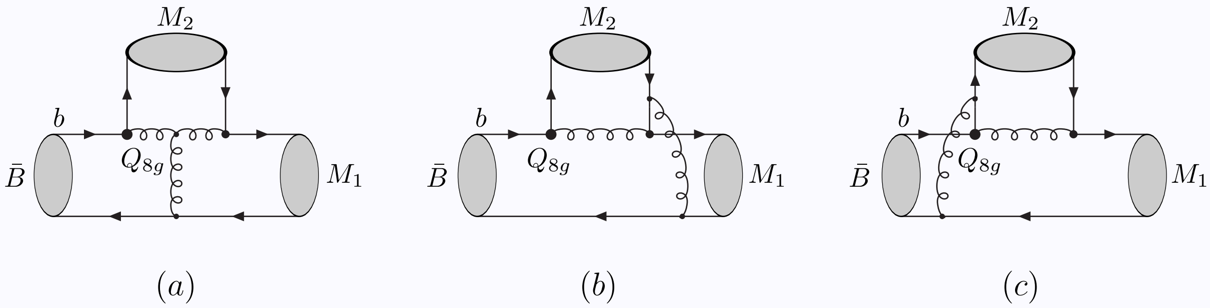
- All "nonfactorizable" power-suppressed contributions, except for the hard-spectator interactions and annihilation terms, have been neglected.
- In calculating the hard-spectator and weak annihilation terms, considering the virtuality of the off-shell gluon, the running coupling constant and the Wilson coefficients should be evaluated at an intermediate scale $\mu_h \sim \sqrt{\Lambda_{\text{QCD}}\, m_b}$, rather than at the scale $\mu \sim m_b$.
- However, the evolution of $C_i(\mu)$ down to μ_h is nontrivial, since the RGE will change below the scale m_b. Here, we do not consider the charm and bottom threshold and evolve the Wilson coefficients in a 5-flavored theory. With this approximation, all logs of the form $\log{(\mu/M_W)}$ have been resumed, while logs of the form $\log{(\mu/m_b)}$ and $\log{(\mu/m_c)}$ are not. Since the latter two terms are never large with $\mu \geq m_b/2$, the approximation would work.

3 Penguin contractions of spectator-scattering amplitudes induced by he $b \to s(d)g^*g^*$ transitions

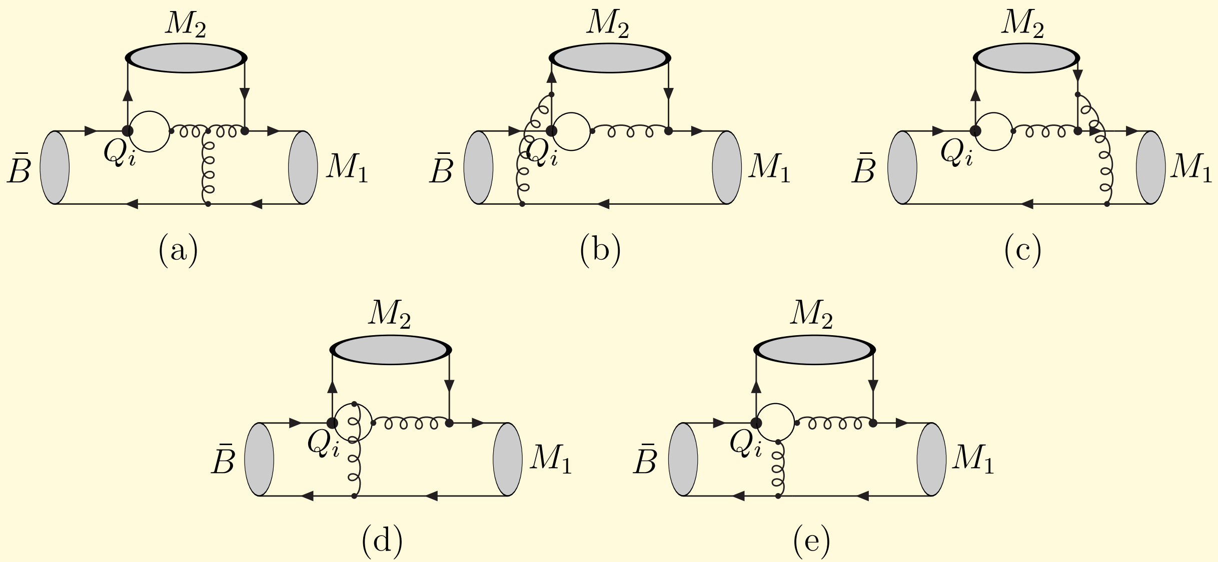
For exclusive hadronic B decays, at the quark level, the $b \to s(d)g^*g^*$ transitions can occur in many different manners. The relevant Feynman diagrams include the following three categories:



Representative Feynman diagrams induced by the $b \to Dg^*g^*$ transitions which are not needed to evaluate. With the operator Q_{8g} replaced by the other operators, the corresponding Feynman diagrams can also be obtained.



Chromo-magnetic dipole operator Q_{8g} contraction contributions induced by the $b \to D g^* g^*$ transitions.



Penguin operator Q_i contraction contributions induced by the $b \to Dg^*g^*$ transitions.

Comments on these higher order penguin contraction contributions:

- Among all the penguin contractions of spectator-scattering amplitudes of order α_s^2, these Feynman diagrams induced by $b \to s(d)g^*g^*$ transitions should be the dominant resources, since they are not genuine two-loop QCD diagrams, and hence there are no additional $\frac{1}{16\pi^2}$ suppression factor in their contributions compared to the genuine two-loop ones of order α_s^2.

- Studying these higher order penguin contraction contributions could be very helpful for our understandings of the dynamical mechanism of hadronic B decays, the higher order perturbative corrections to the rare hadronic B decays within the QCDF formalism.

- In addition, studying these higher order penguin contraction contributions may help us to further constrain possible NP parameter spaces.

- In evaluating these higher order Feynman diagrams, we have adopted the naive dimensional regularization (NDR) scheme and the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme.
- In addition, we have adopted the Feynman gauge for the gluon propagator. In principle, the gauge invariance will be guaranteed when the full set of Feynman diagrams are summed with the external quarks being on-mass-shell. However, we must be careful of the gauge dependence in our calculation, since only a subset $\mathcal{O}(\alpha_s^2)$ Feynman diagrams are calculated.
- After careful checking, we find that each Feynman diagram of our concerns is gauge independent, using the on-shell condition of the external quarks and the gauge invariance of specific Dirac structures of the dipole operator Q_{8g} and the building blocks.

After lengthy and careful calculations, the total contributions of the higher order penguin contractions of spectator-scattering amplitudes induced by $b \to s(d)g^*g^*$ transitions can be written as

$$\begin{aligned}\mathcal{A}'(B \to PV) &= \frac{G_F}{\sqrt{2}} \Big[\sum_{p=u,c} C_1\, \mathcal{A}_{Q_1} + (C_3 - \frac{1}{2} C_9)\, \mathcal{A}_{Q_3} + C_4\, \mathcal{A}_{Q_4} + C_6\, \mathcal{A}_{Q_6} \\ &\quad + C_8\, \mathcal{A}_{Q_8} + C_{10}\, \mathcal{A}_{Q_{10}} + C_{8g}^{\rm eff}\, \mathcal{A}_{Q_{8g}} \Big] , \end{aligned} \tag{8}$$

The total decay amplitude for a given $B \to PV$ decay is then given as

$$\langle PV|\mathcal{H}_{\rm eff}|B\rangle = \mathcal{A}(B \to PV) + \mathcal{A}^{ann}(B \to PV) + \mathcal{A}'(B \to PV) . \tag{9}$$

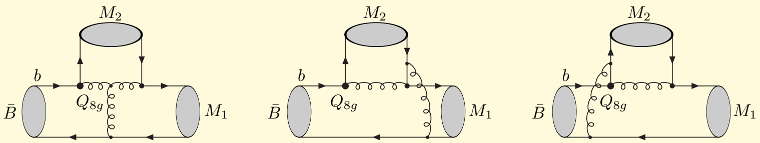
4 Numerical results and Discussions

4.1. Numerical analysis of penguin contraction contributions

We first discuss the relative strength of each Feynman diagram of our concerns.

- Numerical results of each Feynman diagram corresponding to the chromo-magnetic dipole operator Q_{8g} contraction.

	Decay mode	$\Phi_{M_2}\Phi_{M_1}$	$\Phi_{M_2}\Phi_{m_1}$	$\Phi_{m_2}\Phi_{M_1}$	$\Phi_{m_2}\Phi_{m_1}$
Fig.(a)	$B \to PV$	-67.50	-125.76	-9.64	-18.94
	$B \to VP$	-67.50	4.82	34.71	-3.79
Figs.(b+c)	$B \to PV$	-1.50	-3.54	-1.07	-0.42
	$B \to VP$	-1.50	-1.61	1.86	0.42

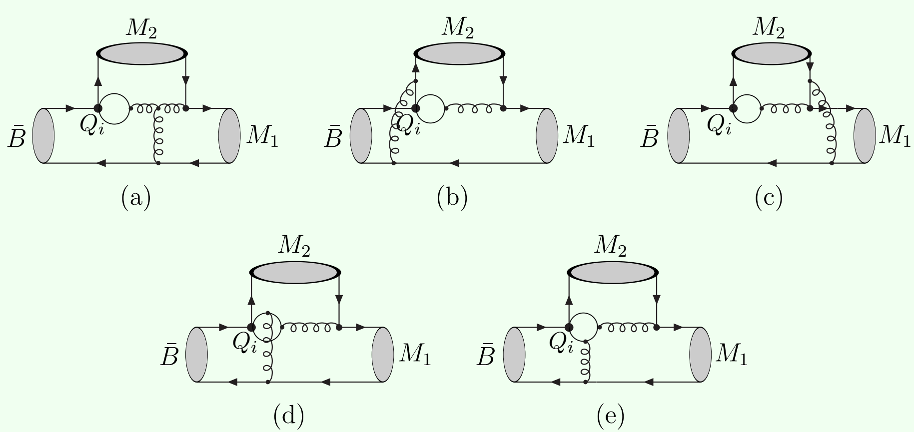


(a) (b) (c)

The main contributions come from Fig.(a), and the other ones play only a minor role. These amplitudes do not have any strong phases.

- Numerical results of each Feynman diagram corresponding to the four-quark operator Q_1^c contraction. Terms involving the twist-three LCDAs are given in unit of the factor r_χ^M.

	modes	$\Phi_{M_2}\Phi_{M_1}$	$\Phi_{M_2}\Phi_{m_1}$	$\Phi_{m_2}\Phi_{M_1}$	$\Phi_{m_2}\Phi_{m_1}$
Fig.(a)	PV	$-1.39-12.65\,i$	$0.17-14.10\,i$	$-0.15+15.38\,i$	$0.12+13.51\,i$
	VP	$-1.39-12.65\,i$	$-0.02+1.28\,i$	$-0.12+11.11\,i$	$-0.01-0.44\,i$
Figs.(b+c)	PV	$-0.01-1.05\,i$	$-0.12-1.21\,i$	$-0.62+0.81\,i$	$-0.18-0.11\,i$
	VP	$-0.01-1.05\,i$	$-0.39-1.25\,i$	$-0.08+0.78\,i$	$-0.10-0.19\,i$
Figs.(d+e)	PV	$-9.03+14.94\,i$	$19.19+28.30\,i$	$4.32-21.29\,i$	$10.82-15.69\,i$
	VP	$-9.03+14.94\,i$	$14.26+9.04\,i$	$0.83-16.78\,i$	$-0.39-3.46\,i$



From the above numerical results, we have the following observations:

- Contributions of Figs.(b) and (c) are generally much smaller than those of the other three ones, and the main contributions come from the diagrams Figs.(d) and (e).
- Although each term labeled by the meson LCDAs in each Feynman diagram has a large imaginary part, and hence a large strong phase, the total strong phase of each Feynman diagram is small due to cancelations among the four terms.
- For each term labeled by the same meson LCDAs, there also exist cancelations between the diagrams Fig.(a) and Figs.(d+e).

Thus, the final total strong phases corresponding to Q_1^c contraction are quite small. The same is true for the other operator contractions.

4.2. Branching ratios of $B \to PV$ decays

- CP-averaged branching ratios (in units of 10^{-6}) of tree-dominated $B \to PV$ decays with $\Delta S = 0$.

Decay mode	NF	$\bar{\mathcal{B}}^f$		$\bar{\mathcal{B}}^{f+a}$		EXP.
		$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \pi^-\rho^0$	$8.76^{+3.56}_{-2.93}$	$8.15^{+3.69}_{-2.86}$	$8.02^{+3.77}_{-2.80}$	$8.13^{+3.53}_{-2.63}$	$8.01^{+3.73}_{-2.58}$	$8.7^{+1.0}_{-1.1}$
	$7.52^{+3.36}_{-2.45}$	$7.45^{+3.42}_{-2.57}$	$7.36^{+3.71}_{-2.67}$	$7.44^{+3.25}_{-2.59}$	$7.36^{+3.60}_{-2.46}$	
$B^- \to \pi^0\rho^-$	$13.91^{+6.21}_{-4.87}$	$13.05^{+6.32}_{-4.53}$	$13.31^{+6.06}_{-4.76}$	$13.22^{+5.94}_{-4.80}$	$13.48^{+6.79}_{-5.05}$	$10.8^{+1.4}_{-1.5}$
	$13.08^{+6.21}_{-4.54}$	$12.82^{+6.32}_{-4.86}$	$13.01^{+6.81}_{-5.16}$	$13.00^{+5.99}_{-4.94}$	$13.20^{+6.12}_{-4.89}$	
$\overline{B}^0 \to \pi^+\rho^-$	$19.78^{+9.88}_{-7.28}$	$19.37^{+9.25}_{-7.62}$	$19.73^{+10.46}_{-7.28}$	$20.34^{+10.20}_{-7.95}$	$20.72^{+9.94}_{-7.85}$	$13.9^{+2.2}_{-2.1}$
	$20.82^{+10.64}_{-7.83}$	$20.22^{+11.10}_{-8.11}$	$20.48^{+11.71}_{-7.65}$	$21.25^{+11.03}_{-8.26}$	$21.52^{+10.22}_{-7.86}$	
$\overline{B}^0 \to \pi^-\rho^+$	$10.72^{+4.61}_{-3.68}$	$10.51^{+4.69}_{-3.55}$	$10.47^{+4.60}_{-3.49}$	$11.15^{+4.71}_{-3.82}$	$11.11^{+4.99}_{-3.75}$	$10.1^{+2.1}_{-1.9}$
	$11.18^{+5.08}_{-3.74}$	$10.90^{+4.71}_{-3.89}$	$10.86^{+4.87}_{-3.92}$	$11.57^{+5.23}_{-4.02}$	$11.52^{+4.99}_{-3.90}$	
$\overline{B}^0 \to \pi^\pm\rho^\mp$	$30.50^{+13.65}_{-10.39}$	$29.88^{+13.22}_{-10.18}$	$30.20^{+13.85}_{-10.52}$	$31.49^{+13.04}_{-10.64}$	$31.83^{+13.82}_{-11.48}$	24.0 ± 2.5
	$32.00^{+14.58}_{-11.12}$	$31.12^{+14.60}_{-10.56}$	$31.34^{+13.82}_{-11.58}$	$32.82^{+14.96}_{-11.82}$	$33.04^{+16.32}_{-11.01}$	
$\overline{B}^0 \to \pi^0\rho^0$	$0.47^{+0.20}_{-0.15}$	$0.40^{+0.35}_{-0.18}$	$0.39^{+0.33}_{-0.15}$	$0.30^{+0.29}_{-0.13}$	$0.30^{+0.27}_{-0.13}$	$1.83^{+0.56}_{-0.55}$
	$0.13^{+0.06}_{-0.04}$	$0.29^{+0.23}_{-0.12}$	$0.29^{+0.24}_{-0.11}$	$0.22^{+0.19}_{-0.08}$	$0.23^{+0.20}_{-0.09}$	
$B^- \to \pi^-\omega$	$7.87^{+3.61}_{-2.57}$	$7.36^{+3.50}_{-2.44}$	$7.47^{+3.80}_{-2.53}$	$7.10^{+3.43}_{-2.62}$	$7.21^{+3.21}_{-2.37}$	6.6 ± 0.6
	$6.96^{+2.94}_{-2.28}$	$6.84^{+3.08}_{-2.39}$	$6.90^{+3.38}_{-2.31}$	$6.54^{+2.89}_{-2.23}$	$6.60^{+3.29}_{-2.28}$	
$\overline{B}^0 \to \pi^0\omega$	$0.01^{+0.03}_{-0.01}$	$0.02^{+0.03}_{-0.01}$	$0.02^{+0.03}_{-0.01}$	$0.005^{+0.015}_{-0.003}$	$0.004^{+0.014}_{-0.003}$	< 1.2
	$0.03^{+0.04}_{-0.02}$	$0.02^{+0.02}_{-0.01}$	$0.02^{+0.03}_{-0.01}$	$0.010^{+0.018}_{-0.007}$	$0.010^{+0.020}_{-0.007}$	

For these decay modes, we have the following general remarks:

- The decays $\overline{B}^0 \to \pi^{\pm}\rho^{\mp}$ and $B^- \to \pi^0\rho^-, \pi^-\rho^0, \pi^-\omega$. Our results are generally consistent with the experimental data within errors. Since these decay channels are dominated by the color-allowed tree amplitudes, both the weak annihilation and the higher order penguin contraction contributions are small. In addition, the main theoretical uncertanities come from the form factors and CKM matrix elements.

- The decays $\overline{B}^0 \to \pi^0\rho^0$ and $\overline{B}^0 \to \pi^0\omega$. Since these decay channels are dominated by the color-suppressed tree amplitudes, their branching ratios are predicted to be very small. The weak annihilation contributions are quite large, while the higher order penguin contraction contributions are small. Besides the form factors and CKM matrix elements, the spectator-scattering amplitudes also cause sizable uncertainties.

- CP-averaged branching ratios (in units of 10^{-6}) of penguin-dominated (the upper six) and annihilation-dominated (the last two) $B \to PV$ decays with $\Delta S = 0$.

Decay mode	NF	$\bar{\mathcal{B}}^f$		$\bar{\mathcal{B}}^{f+a}$		EXP.
		$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to K^- K^{*0}$	$0.15^{+0.07}_{-0.04}$	$0.18^{+0.08}_{-0.07}$	$0.28^{+0.14}_{-0.09}$	$0.23^{+0.11}_{-0.09}$	$0.34^{+0.16}_{-0.11}$	< 5.3
	$0.32^{+0.13}_{-0.11}$	$0.23^{+0.10}_{-0.08}$	$0.33^{+0.15}_{-0.10}$	$0.29^{+0.14}_{-0.10}$	$0.40^{+0.20}_{-0.13}$	
$\overline{B}^0 \to \overline{K}^0 K^{*0}$	$0.14^{+0.06}_{-0.04}$	$0.16^{+0.09}_{-0.06}$	$0.26^{+0.12}_{-0.08}$	$0.20^{+0.10}_{-0.07}$	$0.31^{+0.15}_{-0.10}$	$\ldots$
	$0.29^{+0.14}_{-0.09}$	$0.22^{+0.10}_{-0.08}$	$0.31^{+0.15}_{-0.10}$	$0.26^{+0.10}_{-0.09}$	$0.36^{+0.16}_{-0.11}$	
$B^- \to K^0 K^{*-}$	$0.06^{+0.13}_{-0.04}$	$0.10^{+0.21}_{-0.07}$	$0.10^{+0.20}_{-0.07}$	$0.18^{+0.27}_{-0.10}$	$0.18^{+0.26}_{-0.10}$	$\ldots$
	$0.05^{+0.14}_{-0.04}$	$0.08^{+0.18}_{-0.06}$	$0.07^{+0.17}_{-0.05}$	$0.15^{+0.25}_{-0.09}$	$0.14^{+0.23}_{-0.08}$	
$\overline{B}^0 \to K^0 \overline{K}^{*0}$	$0.06^{+0.12}_{-0.04}$	$0.09^{+0.19}_{-0.06}$	$0.09^{+0.18}_{-0.06}$	$0.18^{+0.26}_{-0.10}$	$0.17^{+0.27}_{-0.09}$	$\ldots$
	$0.04^{+0.14}_{-0.03}$	$0.07^{+0.16}_{-0.05}$	$0.06^{+0.15}_{-0.04}$	$0.15^{+0.25}_{-0.08}$	$0.14^{+0.24}_{-0.08}$	
$B^- \to \pi^- \phi$	≈ 0.001	≈ 0.008	$\ldots$	$\ldots$	$\ldots$	< 0.41
	≈ 0.001	≈ 0.007	$\ldots$	$\ldots$	$\ldots$	
$\overline{B}^0 \to \pi^0 \phi$	≈ 0.0003	≈ 0.004	$\ldots$	$\ldots$	$\ldots$	< 1.0
	≈ 0.0003	≈ 0.003	$\ldots$	$\ldots$	$\ldots$	
$\overline{B}^0 \to K^{*-} K^+$	$\ldots$	$\ldots$	$\ldots$	$0.018^{+0.004}_{-0.004}$	$\ldots$	$\ldots$
	$\ldots$	$\ldots$	$\ldots$	$0.019^{+0.005}_{-0.004}$	$\ldots$	$\ldots$
$\overline{B}^0 \to K^- K^{*+}$	$\ldots$	$\ldots$	$\ldots$	$0.018^{+0.004}_{-0.004}$	$\ldots$	
	$\ldots$	$\ldots$	$\ldots$	$0.019^{+0.005}_{-0.004}$	$\ldots$	

For these decay modes, we have the following general remarks:

- The decays $B^- \to K^- K^{*0}$ and $\overline{B}^0 \to \overline{K}^0 K^{*0}$. These decay channels are dominated by the $b \to d$ penguin amplitudes, and the dominant term is proportional to the coefficient $\alpha_4^p(PV)$. Large interference effects between the two terms are expected and the branching ratios have a strong dependence on the angle γ. The higher order penguin contraction contributions can provide about 60% enhancements. The main theoretical errors originate from the quantity λ_B.

- The decays $B^- \to K^0 K^{*-}$ and $\overline{B}^0 \to K^0 \overline{K}^{*0}$. The dominant contribution is from the coefficient $\alpha_4^p(VP)$, where delicate cancelations between the vector and scalar penguin contributions occur, their branching ratios are relatively small. This also renders the weak annihilation contributions potentially large. The higher order penguin contraction contributions are small. The theoretical errors are mainly due to the strange-quark mass and λ_B.

- The decays $B^- \to \pi^- \phi$ and $\overline{B}^0 \to \pi^0 \phi$. These two decay channels are electro-weak penguin dominated processes. Large "nonfactorizable" contributions dominate these decays, while the theoretical predictions are still quite lower than the experimental upper bounds. The higher order penguin contraction contributions have negligible impact on these decay channels.
- The decays $\overline{B}^0 \to K^+ K^{*-}, K^- K^{*+}$. These two decay channels are pure annihilation processes. The higher order penguin contraction contributions have no impacts on these decay channels. Studying on these decay modes may be helpful to learn more about the strength of annihilation contributions and to provide some useful information about final-state interactions.

- CP-averaged branching ratios (in units of 10^{-6}) of penguin-dominated $B \to PV$ decays with $\Delta S = 1$.

Decay mode	NF	$\bar{\mathcal{B}}^f$		$\bar{\mathcal{B}}^{f+a}$		EXP.
		$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \pi^- \overline{K}^{*0}$	$2.37^{+0.72}_{-0.64}$	$2.60^{+0.95}_{-0.88}$	$4.26^{+1.72}_{-1.21}$	$3.50^{+1.22}_{-1.04}$	$5.39^{+2.01}_{-1.44}$	10.8 ± 0.8
	$4.89^{+1.46}_{-1.28}$	$3.35^{+1.27}_{-1.13}$	$5.01^{+1.81}_{-1.41}$	$4.45^{+1.51}_{-1.36}$	$6.34^{+2.18}_{-1.70}$	
$B^- \to \pi^0 K^{*-}$	$1.82^{+0.76}_{-0.54}$	$1.88^{+0.79}_{-0.56}$	$2.73^{+1.23}_{-0.81}$	$2.33^{+0.96}_{-0.69}$	$3.29^{+1.31}_{-0.89}$	6.9 ± 2.3
	$3.03^{+1.15}_{-0.88}$	$2.21^{+0.87}_{-0.74}$	$3.05^{+1.25}_{-0.89}$	$2.75^{+1.08}_{-0.79}$	$3.70^{+1.36}_{-1.01}$	
$\overline{B}^0 \to \pi^+ K^{*-}$	$1.84^{+0.90}_{-0.67}$	$1.92^{+0.89}_{-0.72}$	$3.04^{+1.64}_{-1.04}$	$2.47^{+1.08}_{-0.82}$	$3.78^{+1.84}_{-1.34}$	$11.7^{+1.5}_{-1.4}$
	$3.40^{+1.49}_{-1.11}$	$2.32^{+1.12}_{-0.84}$	$3.43^{+1.67}_{-1.13}$	$2.99^{+1.31}_{-0.96}$	$4.30^{+2.09}_{-1.44}$	
$\overline{B}^0 \to \pi^0 \overline{K}^{*0}$	$0.49^{+0.27}_{-0.20}$	$0.53^{+0.35}_{-0.26}$	$1.08^{+0.77}_{-0.46}$	$0.80^{+0.42}_{-0.33}$	$1.45^{+0.86}_{-0.56}$	1.7 ± 0.8
	$1.24^{+0.56}_{-0.46}$	$0.73^{+0.50}_{-0.35}$	$1.28^{+0.73}_{-0.50}$	$1.07^{+0.56}_{-0.43}$	$1.72^{+0.91}_{-0.65}$	
$B^- \to K^- \phi$	$3.71^{+1.18}_{-1.00}$	$2.73^{+1.33}_{-1.20}$	$5.06^{+2.01}_{-1.48}$	$4.04^{+1.58}_{-1.48}$	$6.77^{+2.78}_{-1.74}$	$9.03^{+0.65}_{-0.63}$
	$10.17^{+3.21}_{-3.23}$	$3.90^{+1.93}_{-1.69}$	$6.32^{+2.07}_{-1.77}$	$5.59^{+2.23}_{-2.11}$	$8.42^{+2.67}_{-2.22}$	
$\overline{B}^0 \to \overline{K}^0 \phi$	$3.45^{+1.10}_{-0.93}$	$2.53^{+1.20}_{-1.11}$	$4.70^{+1.86}_{-1.37}$	$3.67^{+1.50}_{-1.37}$	$6.19^{+2.40}_{-1.69}$	$8.3^{+1.2}_{-1.0}$
	$9.46^{+3.01}_{-2.59}$	$3.63^{+1.81}_{-1.61}$	$5.88^{+2.10}_{-1.67}$	$5.09^{+2.10}_{-1.87}$	$7.70^{+2.55}_{-2.14}$	

- CP-averaged branching ratios (in units of 10^{-6}) of penguin-dominated $B \to PV$ decays with $\Delta S = 1$. (continued)

Decay mode	NF	$\bar{\mathcal{B}}^f$		$\bar{\mathcal{B}}^{f+a}$		EXP.
		$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \overline{K}^0 \rho^-$	$1.05^{+2.12}_{-0.73}$	$1.74^{+3.09}_{-1.16}$	$1.65^{+3.10}_{-1.08}$	$3.18^{+4.42}_{-1.85}$	$3.05^{+3.94}_{-1.73}$	< 48
	$0.76^{+2.17}_{-0.63}$	$1.36^{+2.99}_{-0.97}$	$1.20^{+2.69}_{-0.86}$	$2.73^{+3.77}_{-1.58}$	$2.49^{+3.83}_{-1.47}$	
$B^- \to K^- \rho^0$	$0.77^{+1.06}_{-0.35}$	$0.99^{+1.70}_{-0.59}$	$0.96^{+1.69}_{-0.56}$	$1.56^{+2.38}_{-0.95}$	$1.51^{+2.24}_{-0.95}$	$4.23^{+0.56}_{-0.57}$
	$0.58^{+1.11}_{-0.26}$	$0.78^{+1.56}_{-0.43}$	$0.72^{+1.35}_{-0.36}$	$1.28^{+2.10}_{-0.78}$	$1.19^{+2.12}_{-0.70}$	
$\overline{B}^0 \to K^- \rho^+$	$2.50^{+3.17}_{-1.36}$	$3.44^{+4.20}_{-1.91}$	$3.31^{+4.09}_{-1.81}$	$5.27^{+5.29}_{-2.67}$	$5.11^{+5.18}_{-2.55}$	$9.9^{+1.6}_{-1.5}$
	$2.28^{+3.33}_{-1.33}$	$3.04^{+3.66}_{-1.69}$	$2.81^{+3.77}_{-1.54}$	$4.86^{+5.19}_{-2.42}$	$4.55^{+5.00}_{-2.32}$	
$\overline{B}^0 \to \overline{K}^0 \rho^0$	$1.42^{+1.59}_{-0.72}$	$1.98^{+2.13}_{-1.03}$	$1.90^{+2.12}_{-0.97}$	$3.03^{+3.01}_{-1.35}$	$2.94^{+2.68}_{-1.39}$	5.1 ± 1.6
	$1.32^{+1.79}_{-0.76}$	$1.80^{+2.17}_{-0.94}$	$1.66^{+1.97}_{-0.95}$	$2.88^{+2.61}_{-1.35}$	$2.70^{+2.59}_{-1.27}$	
$B^- \to K^- \omega$	$0.89^{+1.18}_{-0.48}$	$2.16^{+2.33}_{-1.12}$	$2.10^{+2.55}_{-1.11}$	$3.07^{+3.01}_{-1.49}$	$2.99^{+3.07}_{-1.44}$	6.5 ± 0.6
	$0.40^{+0.87}_{-0.13}$	$1.75^{+2.15}_{-0.97}$	$1.65^{+2.27}_{-0.94}$	$2.61^{+3.20}_{-1.42}$	$2.47^{+3.25}_{-1.29}$	
$\overline{B}^0 \to \overline{K}^0 \omega$	$0.17^{+0.66}_{-0.15}$	$1.03^{+1.74}_{-0.68}$	$0.99^{+1.67}_{-0.66}$	$1.78^{+2.45}_{-1.00}$	$1.72^{+2.26}_{-0.96}$	4.7 ± 0.6
	$0.03^{+0.29}_{-0.03}$	$0.76^{+1.49}_{-0.52}$	$0.69^{+1.45}_{-0.47}$	$1.43^{+2.16}_{-0.83}$	$1.33^{+2.09}_{-0.82}$	

For these decay modes, we have the following general remarks:

- The decays $B \to \pi K^*$ and $B \to \phi K$. Our central results are still lower than the experimental data. The dominant contribution is the coefficient $\alpha_4^p(PV)$. The higher order penguin contraction contributions can give enhancements by about $40\% \sim 90\%$. Large interference effects between the tree and penguin amplitudes in $\overline{B}^0 \to \pi^+ K^{*-}$ and $B^- \to \pi^0 K^{*-}$, are expected, thus possible to gain information on the angle γ. The main theoretical errors are due to the CKM elements, form factors, and λ_B.

- The decays $B \to K\rho$ and $B \to K\omega$. The dominant term is the coefficient $\alpha_4^p(VP)$. Because of the destructive interference between the vector and the scalar penguin contributions, these branching ratios are much smaller than their $B \to PP$ counterparts. The higher order penguin contraction contributions are quite small, and tend to decrease the NLO results. The main theoretical errors are from the strange-quark mass and form factors.

4.3. Direct CP-violating asymmetries of $B \to PV$ decays

- Direct CP-violating asymmetries (in units of 10^{-2}) for tree-dominated $B \to PV$ decays with $\Delta S = 0$.

Decay mode	$\mathcal{A}_{CP}^{f}$		$\mathcal{A}_{CP}^{f+a}$		EXP.
	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \pi^-\rho^0$	$3.25^{+1.98}_{-1.27}$	$5.26^{+3.62}_{-2.06}$	$3.62^{+2.29}_{-1.43}$	$5.64^{+3.58}_{-2.13}$	-7^{+12}_{-13}
	$2.83^{+2.35}_{-1.33}$	$4.02^{+3.04}_{-1.58}$	$3.39^{+2.36}_{-1.53}$	$4.58^{+2.78}_{-1.78}$	
$B^- \to \pi^0\rho^-$	$-2.41^{+0.81}_{-1.61}$	$-3.69^{+1.39}_{-2.48}$	$-2.63^{+0.83}_{-1.63}$	$-3.88^{+1.37}_{-2.52}$	1 ± 11
	$-1.74^{+0.68}_{-1.54}$	$-2.49^{+0.94}_{-1.84}$	$-2.03^{+0.76}_{-1.70}$	$-2.76^{+0.95}_{-1.87}$	
$\overline{B}^0 \to \pi^+\rho^-$	$-1.05^{+0.12}_{-0.19}$	$-2.65^{+0.92}_{-1.85}$	$-1.03^{+0.12}_{-0.17}$	$-2.57^{+0.80}_{-1.82}$	-15 ± 9
	$-0.68^{+0.08}_{-0.11}$	$-1.68^{+0.45}_{-1.03}$	$-0.65^{+0.07}_{-0.11}$	$-1.62^{+0.44}_{-0.89}$	
$\overline{B}^0 \to \pi^-\rho^+$	$0.40^{+0.64}_{-0.37}$	$-0.03^{+0.64}_{-0.60}$	$0.31^{+0.58}_{-0.37}$	$-0.13^{+0.64}_{-0.53}$	-47^{+13}_{-14}
	$-0.76^{+0.23}_{-0.27}$	$-1.36^{+0.41}_{-0.65}$	$-0.88^{+0.23}_{-0.29}$	$-1.49^{+0.40}_{-0.64}$	
$\overline{B}^0 \to \pi^0\rho^0$	$-5.64^{+9.80}_{-17.89}$	$5.92^{+10.14}_{-17.18}$	$-13.49^{+11.83}_{-20.61}$	$-0.22^{+12.35}_{-23.54}$	-49^{+70}_{-83}
	$-4.42^{+19.18}_{-28.38}$	$10.58^{+18.83}_{-28.48}$	$-19.13^{+18.98}_{-32.25}$	$-1.68^{+21.36}_{-34.52}$	
$B^- \to \pi^-\omega$	$-1.95^{+1.54}_{-2.03}$	$-4.49^{+1.68}_{-2.34}$	$-1.84^{+1.58}_{-2.09}$	$-4.45^{+1.66}_{-2.16}$	-4 ± 8
	$-4.46^{+2.11}_{-3.14}$	$-6.66^{+2.38}_{-3.37}$	$-4.36^{+2.10}_{-3.09}$	$-6.64^{+2.42}_{-3.21}$	

- Direct CP-violating asymmetries (in units of 10^{-2}) for penguin-dominated $B \to PV$ decays with $\Delta S = 0$.

Decay mode	$\mathcal{A}_{CP}^{f}$		$\mathcal{A}_{CP}^{f+a}$		EXP.
	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to K^- K^{*0}$	$-36.28^{+5.04}_{-5.51}$	$-19.29^{+8.89}_{-6.15}$	$-31.08^{+4.37}_{-4.67}$	$-15.34^{+8.74}_{-6.47}$	...
	$-42.06^{+5.68}_{-6.38}$	$-28.33^{+6.84}_{-5.54}$	$-36.92^{+5.40}_{-5.29}$	$-24.27^{+6.78}_{-5.82}$	
$\overline{B}^0 \to \overline{K}^0 K^{*0}$	$-36.27^{+5.02}_{-5.66}$	$-19.29^{+8.34}_{-6.48}$	$-32.72^{+4.74}_{-4.82}$	$-17.56^{+7.65}_{-5.57}$	...
	$-42.06^{+5.43}_{-6.50}$	$-28.33^{+6.91}_{-5.56}$	$-38.64^{+5.15}_{-5.46}$	$-26.25^{+6.10}_{-6.04}$	
$B^- \to K^0 K^{*-}$	$-12.64^{+4.49}_{-4.14}$	$-22.25^{+4.35}_{-7.40}$	$-9.41^{+5.03}_{-4.82}$	$-15.93^{+4.95}_{-4.54}$	...
	$-2.96^{+8.53}_{-6.64}$	$-18.26^{+5.22}_{-9.82}$	$0.18^{+10.23}_{-7.16}$	$-9.17^{+8.89}_{-6.79}$	
$\overline{B}^0 \to K^0 \overline{K}^{*0}$	$-12.64^{+4.60}_{-4.00}$	$-22.25^{+4.24}_{-8.09}$	$-9.25^{+4.55}_{-4.78}$	$-16.25^{+4.90}_{-4.25}$	...
	$-2.96^{+8.64}_{-6.76}$	$-18.26^{+5.59}_{-8.60}$	$-1.76^{+6.45}_{-5.73}$	$-12.22^{+5.90}_{-6.34}$	

- Direct CP-violating asymmetries (in units of 10^{-2}) for $B \to PV$ decays with $\Delta S = 1$.

Decay mode	$\mathcal{A}_{CP}^{f}$		$\mathcal{A}_{CP}^{f+a}$		EXP.
	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \pi^- \overline{K}^{*0}$	$1.49^{+0.23}_{-0.14}$	$0.76^{+0.26}_{-0.34}$	$1.22^{+0.14}_{-0.13}$	$0.57^{+0.26}_{-0.34}$	-9.3 ± 6.0
	$1.77^{+0.24}_{-0.17}$	$1.14^{+0.21}_{-0.24}$	$1.47^{+0.17}_{-0.15}$	$0.93^{+0.21}_{-0.26}$	
$B^- \to \pi^0 K^{*-}$	$14.03^{+2.88}_{-2.44}$	$18.21^{+5.43}_{-4.15}$	$11.98^{+2.46}_{-2.10}$	$15.48^{+4.69}_{-3.59}$	4 ± 29
	$13.09^{+3.48}_{-2.66}$	$14.85^{+3.47}_{-3.07}$	$11.27^{+2.74}_{-2.40}$	$12.72^{+2.74}_{-2.43}$	
$\overline{B}^0 \to \pi^+ K^{*-}$	$9.14^{+1.51}_{-1.34}$	$17.18^{+6.39}_{-4.76}$	$7.11^{+1.31}_{-1.24}$	$13.75^{+5.50}_{-4.06}$	-5 ± 14
	$3.89^{+0.65}_{-0.59}$	$9.16^{+2.87}_{-2.09}$	$2.86^{+0.52}_{-0.49}$	$7.16^{+1.93}_{-1.46}$	
$\overline{B}^0 \to \pi^0 \overline{K}^{*0}$	$-11.58^{+4.15}_{-8.58}$	$-9.94^{+3.14}_{-4.69}$	$-9.20^{+2.79}_{-5.00}$	$-8.34^{+2.64}_{-3.77}$	-1^{+27}_{-26}
	$-12.14^{+4.04}_{-7.46}$	$-10.06^{+3.09}_{-4.31}$	$-9.97^{+3.36}_{-4.79}$	$-8.60^{+2.47}_{-3.68}$	
$B^- \to K^- \phi$	$2.08^{+0.53}_{-0.27}$	$1.07^{+0.32}_{-0.37}$	$1.61^{+0.23}_{-0.18}$	$0.78^{+0.30}_{-0.39}$	3.7 ± 5.0
	$2.33^{+0.56}_{-0.31}$	$1.49^{+0.21}_{-0.22}$	$1.84^{+0.27}_{-0.20}$	$1.17^{+0.23}_{-0.23}$	
$\overline{B}^0 \to \overline{K}^0 \phi$	$2.08^{+0.50}_{-0.27}$	$1.07^{+0.33}_{-0.39}$	$1.72^{+0.27}_{-0.19}$	$0.92^{+0.25}_{-0.39}$	9 ± 14
	$2.33^{+0.58}_{-0.29}$	$1.49^{+0.20}_{-0.23}$	$1.96^{+0.33}_{-0.21}$	$1.30^{+0.19}_{-0.25}$	

- Direct CP-violating asymmetries (in units of 10^{-2}) for $B \to PV$ decays with $\Delta S = 1$ (continued).

Decay mode	$\mathcal{A}_{CP}^{f}$		$\mathcal{A}_{CP}^{f+a}$		EXP.
	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	
$B^- \to \overline{K}^0\rho^-$	$0.49^{+0.14}_{-0.17}$	$0.93^{+0.34}_{-0.15}$	$0.37^{+0.17}_{-0.21}$	$0.67^{+0.20}_{-0.18}$	$\dots$
	$0.11^{+0.25}_{-0.32}$	$0.80^{+0.41}_{-0.20}$	$-0.02^{+0.29}_{-0.40}$	$0.41^{+0.28}_{-0.36}$	
$B^- \to K^-\rho^0$	$-7.99^{+11.58}_{-5.17}$	$-3.62^{+17.39}_{-6.87}$	$-7.32^{+4.63}_{-3.55}$	$-4.55^{+8.50}_{-4.26}$	31^{+12}_{-11}
	$5.88^{+27.17}_{-10.73}$	$15.31^{+33.85}_{-16.23}$	$-0.38^{+13.62}_{-5.64}$	$5.17^{+23.92}_{-8.71}$	
$\overline{B}^0 \to K^-\rho^+$	$-1.76^{+1.64}_{-0.87}$	$0.24^{+4.86}_{-1.84}$	$-0.91^{+1.21}_{-0.85}$	$0.46^{+2.99}_{-1.42}$	17^{+15}_{-16}
	$4.12^{+4.76}_{-2.50}$	$7.89^{+10.46}_{-4.88}$	$3.02^{+3.08}_{-1.67}$	$5.44^{+6.24}_{-3.07}$	
$\overline{B}^0 \to \overline{K}^0\rho^0$	$9.58^{+3.69}_{-3.24}$	$9.73^{+3.86}_{-3.29}$	$7.65^{+2.85}_{-2.30}$	$7.78^{+2.67}_{-2.45}$	$\dots$
	$12.36^{+5.78}_{-4.30}$	$12.91^{+5.89}_{-4.81}$	$9.81^{+3.63}_{-3.16}$	$10.23^{+4.29}_{-3.46}$	
$B^- \to K^-\omega$	$-4.71^{+2.93}_{-2.41}$	$-2.85^{+4.26}_{-3.31}$	$-4.35^{+2.05}_{-1.93}$	$-3.04^{+2.92}_{-2.35}$	2 ± 7
	$4.75^{+13.57}_{-5.57}$	$8.69^{+16.81}_{-7.30}$	$1.39^{+6.29}_{-3.35}$	$3.94^{+9.10}_{-4.60}$	
$\overline{B}^0 \to \overline{K}^0\omega$	$-9.65^{+4.10}_{-5.65}$	$-8.90^{+3.91}_{-5.41}$	$-7.61^{+2.96}_{-4.62}$	$-7.13^{+2.69}_{-3.99}$	44 ± 23
	$-12.85^{+5.95}_{-6.22}$	$-11.61^{+5.50}_{-5.40}$	$-10.55^{+4.54}_{-7.83}$	$-9.94^{+4.27}_{-6.60}$	

From these numerical results, we have the following general remarks:

- Since the strong phases are suppressed by α_s and/or $\Lambda_{\rm QCD}/m_b$ within the QCDF formalism, the direct CP asymmetries for most $B \to PV$ decays are predicted to be typically small within this approach.

- Due to large cancelations among the strong phases associated with the higher order penguin contraction contributions, the new higher order contributions have only small effects on the direct CP asymmetries.

- However, for $b \to d$ penguin dominated $B \to K\bar{K}^*$ decays, since $\alpha_4^c \approx \alpha_4^u$ and $|V_{ub}^* V_{ud}| \approx |V_{cb}^* V_{cd}|$, large direct CP asymmetries are predicted. In addition, due to large interference effects between the tree and penguin amplitudes, the direct CP asymmetry of $B^- \to \pi^0 K^{*-}$ decay is also predicted to be large.

- Both the higher order penguin contraction and the weak annihilation contributions have significant impacts on the direct CP asymmetry of $\overline{B}^0 \to \pi^0\rho^0$ decay, due to the delicate cancelations among the competing terms, making these sub-leading contributing terms potentially large.

- The higher order penguin contraction contributions to the direct CP asymmetries of $\overline{B}^0 \to \pi^+\rho^-$, $B^- \to \pi^-\omega$, $\overline{B}^0 \to \pi^-\rho^+$, and $\overline{B}^0 \to K^-\rho^+$ decays are also quite large, increasing the direct CP asymmetries of the former two, while decreasing those of the latter two by the same magnitude.

- Although the uncertainties from various input parameters are reduced to some extent, the renormalization scale dependence of the direct CP asymmetries for some decay modes, such as $B^- \to K^-\omega$ and $B^- \to K^-\rho^0$ decays, are still large. This is due to the fact that the imaginary parts of the coefficients α_i generally have a larger scale dependence.

4.4. Ratios between the branching fractions of $B \to \pi K^*$, $K\rho$ decays

Since theoretical uncertainties in the branching ratios can be largely reduced by taking ratios among them, we define the following three ratios, just like the ones defined for $B \to \pi K$ decays.

$$R(\pi K^*) \equiv \frac{\tau_{B_u}}{\tau_{B_d}} \frac{\bar{\mathcal{B}}(\overline{B}^0 \to \pi^+ K^{*-})}{\bar{\mathcal{B}}(B^- \to \pi^- \overline{K}^{*0})}, \tag{10}$$

$$R_c(\pi K^*) \equiv 2 \frac{\bar{\mathcal{B}}(B^- \to \pi^0 K^{*-})}{\bar{\mathcal{B}}(B^- \to \pi^- \overline{K}^{*0})}, \tag{11}$$

$$R_n(\pi K^*) \equiv \frac{1}{2} \frac{\bar{\mathcal{B}}(\overline{B}^0 \to \pi^+ K^{*-})}{\bar{\mathcal{B}}(\overline{B}^0 \to \pi^0 \overline{K}^{*0})}. \tag{12}$$

With $\pi(K^*)$ meson replaced by $\rho(K)$ meson, we can get another three similar ratios for $B \to K\rho$ decays. These ratios should be more appropriate to derive information on the weak phase angle γ, as well as the relative strength of tree and penguin contributions than branching ratios.

- Ratios among CP-averaged branching fractions of $B \to \pi K^*, K\rho$ decays.

	NF	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s+\alpha_s^2)$	EXP.
$R(\pi K^*)$	$0.84^{+0.16}_{-0.14}$	$0.80^{+0.18}_{-0.14}$	$0.77^{+0.14}_{-0.11}$	$0.76^{+0.15}_{-0.12}$	$0.76^{+0.12}_{-0.11}$	1.18 ± 0.17
	$0.75^{+0.11}_{-0.09}$	$0.74^{+0.16}_{-0.11}$	$0.74^{+0.11}_{-0.11}$	$0.72^{+0.15}_{-0.10}$	$0.73^{+0.11}_{-0.10}$	
$R_c(\pi K^*)$	$1.53^{+0.45}_{-0.31}$	$1.45^{+0.49}_{-0.31}$	$1.28^{+0.29}_{-0.22}$	$1.33^{+0.37}_{-0.26}$	$1.22^{+0.25}_{-0.20}$	1.28 ± 0.44
	$1.24^{+0.27}_{-0.21}$	$1.32^{+0.45}_{-0.27}$	$1.22^{+0.28}_{-0.20}$	$1.24^{+0.32}_{-0.23}$	$1.17^{+0.23}_{-0.19}$	
$R_n(\pi K^*)$	$1.87^{+0.94}_{-0.53}$	$1.80^{+1.14}_{-0.53}$	$1.41^{+0.51}_{-0.32}$	$1.54^{+0.66}_{-0.42}$	$1.31^{+0.42}_{-0.26}$	3.44 ± 1.68
	$1.37^{+0.48}_{-0.29}$	$1.58^{+0.80}_{-0.44}$	$1.33^{+0.50}_{-0.28}$	$1.40^{+0.53}_{-0.34}$	$1.25^{+0.43}_{-0.24}$	
$R(\rho K)$	$2.55^{+2.45}_{-0.94}$	$2.12^{+1.54}_{-0.67}$	$2.17^{+1.73}_{-0.72}$	$1.78^{+0.85}_{-0.41}$	$1.80^{+0.87}_{-0.41}$	> 0.22
	$3.20^{+6.80}_{-1.48}$	$2.41^{+2.62}_{-0.85}$	$2.53^{+2.78}_{-0.93}$	$1.91^{+1.04}_{-0.50}$	$1.97^{+1.16}_{-0.55}$	
$R_c(\rho K)$	$1.47^{+1.96}_{-0.66}$	$1.14^{+0.99}_{-0.41}$	$1.16^{+1.12}_{-0.43}$	$0.98^{+0.53}_{-0.26}$	$0.99^{+0.56}_{-0.30}$	> 0.18
	$1.52^{+4.78}_{-0.80}$	$1.14^{+1.49}_{-0.49}$	$1.21^{+1.76}_{-0.55}$	$0.94^{+0.60}_{-0.31}$	$0.95^{+0.75}_{-0.29}$	
$R_n(\rho K)$	$0.88^{+0.44}_{-0.26}$	$0.87^{+0.34}_{-0.23}$	$0.87^{+0.35}_{-0.24}$	$0.87^{+0.25}_{-0.20}$	$0.87^{+0.26}_{-0.21}$	0.97 ± 0.34
	$0.87^{+0.52}_{-0.26}$	$0.84^{+0.39}_{-0.23}$	$0.85^{+0.40}_{-0.27}$	$0.84^{+0.26}_{-0.20}$	$0.84^{+0.26}_{-0.21}$	

Main observations about these ratios are:

- Our theoretical predictions for most of these ratios are in agreement with the data, considering the large uncertainties in the experimental data.

- The current data indicate that $R_n(\pi K^*)$ is somewhat larger that $R_c(\pi K^*)$, but with large errors in the former. Due to the insufficient data on the $K\rho$ modes, direct experimental comparison between $R_c(\rho K)$ and $R_n(\rho K)$ is not feasible currently.
- Theoretically, differences between the two ratios R_c and R_n for both πK^* and $K\rho$ modes arise mainly from the EW penguin coefficient $\alpha^p_{3,ew}$ and the color-suppressed tree coefficient α_2, both are predicted to be small here. So, the ratios R_c and R_n are expected to be approximately equal within the SM. However, due to delicate cancelations among various competing terms, they are strongly affected by sub-leading contributions. After including the annihilation contributions, R_c and R_n tend to be approximately equal.
- These ratios remain nearly unaffected even with these new higher order penguin contributions included, because their contributions to the decays in the same ratio are similar in nature, and hence eliminated.

5 Conclusions

Charmless $B \to PV$ (with $P = (\pi, K)$, and $V = (\rho, K^*, \omega, \phi)$) decays have been re-analyzed within the QCDF framework, taking into account the penguin contractions of spectator-scattering amplitudes induced by the $b \to s(d)g^*g^*$ transitions, which are of order α_s^2.

Although the theoretical results presented here still have large uncertainties, the higher order penguin contractions of spectator-scattering amplitudes induced by the $b \to s(d)g^*g^*$ transitions, have been shown to be very important for exclusive $B \to PV$ decays, particularly for those penguin-dominated ones.

Further detailed analysis of these higher order corrections are very promising and interesting.

Thanks to all!