



# 含 $W'$ 的电弱手征拉氏量

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工作的动机

含 $W'^{\pm}, Z'$ 的电弱手征拉氏量

$W - W'$ 混合、费米子的混合

$\Delta m_K, |\epsilon_K|, \Delta m_{B_d}, \Delta m_{B_s}$



## 工作的动机

- LHC开始运行！期待发现新的粒子， $\Rightarrow$ 超出标准模型的新物理！
- 我们关心新粒子是带电的矢量粒子的情形！
- 理论的幺正性要求它是自发破缺的非阿贝尔规范理论中的规范粒子.
- 最小的非阿贝尔规范对称性是  $SU(2)$ ，产生规范粒子  $\underline{W'^{\pm}, Z'}$
- 构造含  $W'^{\pm}, Z'$  的电弱手征拉氏量，进行模型无关和相关的研究
- 为改善理论的幺正性，还得加上一个中性Higgs粒子  $h$
- 期待研究的物理情形：最轻的新粒子是  $h$  和  $W'^{\pm}$

# 含 $W'^{\pm}, Z'$ 的电弱手征拉氏量

- 粒子内容: ♠ 标准模型中所有已被发现的粒子  
♣  $h, W'^{\pm}, Z'$ 及其对应的三个**Goldstone**玻色子
- 对称性的实现方式:  $SU(2)_1 \otimes SU(2)_2 \otimes U(1) \rightarrow U(1)_{\text{em}}$
- $W'^{\pm}, Z'$ 耦合夸克 $q - \bar{q}$ 、轻子 $l - \bar{l}$ : 
 LR : Left – right symmetric  
 LP : Leptophobic  
 HP : Hadrophobic  
 FP : Fermionphobic  
 UN : Ununified  
 NU : Non – universal
- 不讨论**leptoquark**:  $W'$ 将夸克耦合到轻子



# 不同的费米子表示安排

Fields/Models	LR	LP	HP	FP	UN	NU
$q_{\alpha L} = \begin{pmatrix} u_{\alpha L} \\ d_{\alpha L} \end{pmatrix}$	$(2, 1, \frac{1}{6})$	$(2, 1, \frac{1}{6})$	$(2, 1, \frac{1}{6})$	$(2, 1, \frac{1}{6})$	$(1, 2, \frac{1}{6})$	$(2, 1, \frac{1}{6})\delta_{\alpha\alpha_1} + (1, 2, \frac{1}{6})\delta_{\alpha\alpha_2}$
$q_{\alpha R} = \begin{pmatrix} u_{\alpha R} \\ d_{\alpha R} \end{pmatrix}$	$(1, 2, \frac{1}{6})$	$(1, 2, \frac{1}{6})$	$(1, 1, \frac{2}{3})$ $(1, 1, -\frac{1}{3})$			
$l_{\alpha L} = \begin{pmatrix} \nu_{\alpha L} \\ e_{\alpha L}^- \end{pmatrix}$	$(2, 1, -\frac{1}{2})$	$(2, 1, -\frac{1}{2})$	$(2, 1, -\frac{1}{2})$	$(2, 1, -\frac{1}{2})$	$(2, 1, -\frac{1}{2})$	$(2, 1, -\frac{1}{2})\delta_{\alpha\alpha_1} + (1, 2, -\frac{1}{2})\delta_{\alpha\alpha_2}$
$l_{\alpha R} = \begin{pmatrix} \nu_{\alpha R} \\ e_{\alpha R}^- \end{pmatrix}$	$(1, 2, -\frac{1}{2})$	$(1, 1, 0)$ $(1, 1, -1)$	$(1, 2, -\frac{1}{2})$	$(1, 1, 0)$ $(1, 1, -1)$	$(1, 1, 0)$ $(1, 1, -1)$	$(1, 1, 0)$ $(1, 1, -1)$



Y.Zhang, S-Z.Wang, F-J.Ge, Q.Wang, Phys. Lett. B653,259(2007)

$$\overline{W}_{i,\mu\nu} \equiv U_i^\dagger g_i W_{i,\mu\nu} U_i \quad X_i^\mu \equiv U_i^\dagger (D^\mu U_i) \quad D_\mu U_i = \partial_\mu U_i + ig_i \frac{\tau^a}{2} W_{i,\mu}^a U_i - ig U_i \frac{\tau_3}{2} B_\mu$$

$$\begin{aligned} \mathcal{L}_{\text{EWCL}}^{\text{boson}} = & -V(h) + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4}f_1^2 \text{tr}(X_{1,\mu} X_1^\mu) - \frac{1}{4}f_2^2 \text{tr}(X_{2,\mu} X_2^\mu) + \frac{1}{2}\kappa f_1 f_2 \text{tr}(X_1^\mu X_2^\mu) \\ & + \frac{1}{4}\beta_{1,1} f_1^2 [\text{tr}(\tau^3 X_{1,\mu})]^2 + \frac{1}{4}\beta_{2,1} f_2^2 [\text{tr}(\tau^3 X_{2,\mu})]^2 + \frac{1}{2}\tilde{\beta}_1 f_1 f_2 [\text{tr}(\tau^3 X_{1,\mu})][\text{tr}(\tau^3 X_2^\mu)] \\ & + \mathcal{L}_K + \mathcal{L}_1 + \mathcal{L}_{H1} + \mathcal{L}_2 + \mathcal{L}_{H2} + \mathcal{L}_C + O(p^6) \end{aligned}$$

$$\mathcal{L}_K = -\frac{1}{4}W_{1,\mu\nu}^a W_1^{\mu\nu,a} - \frac{1}{4}W_{2,\mu\nu}^a W_2^{\mu\nu,a} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$$

$$\begin{aligned} \mathcal{L}_i = & \frac{1}{2}\alpha_{i,1}g B_{\mu\nu} \text{tr}(\tau^3 \overline{W}_i^{\mu\nu}) + i\alpha_{i,2}g B_{\mu\nu} \text{tr}(\tau^3 X_i^\mu X_i^\nu) + 2i\alpha_{i,3} \text{tr}(\overline{W}_{i,\mu\nu} X_i^\mu X_i^\nu) + \alpha_{i,4}[\text{tr}(X_{i,\mu} X_{i,\nu})]^2 \\ & + \alpha_{i,5}[\text{tr}(X_{i,\mu}^2)]^2 + \alpha_{i,6} \text{tr}(X_{i,\mu} X_{i,\nu}) \text{tr}(\tau^3 X_i^\mu) \text{tr}(\tau^3 X_i^\nu) + \alpha_{i,7} \text{tr}(X_{i,\mu}^2) [\text{tr}(\tau^3 X_{i,\nu})]^2 \\ & + \frac{1}{4}\alpha_{i,8}[\text{tr}(\tau^3 \overline{W}_{i,\mu\nu})]^2 + i\alpha_{i,9} \text{tr}(\tau^3 \overline{W}_{i,\mu\nu}) \text{tr}(\tau^3 X_i^\mu X_i^\nu) + \frac{1}{2}\alpha_{i,10}[\text{tr}(\tau^3 X_{i,\mu}) \text{tr}(\tau^3 X_{i,\nu})]^2 \\ & + \alpha_{i,11}\epsilon^{\mu\nu\rho\lambda} \text{tr}(\tau^3 X_{i,\mu}) \text{tr}(X_{i,\nu} \overline{W}_{i,\rho\lambda}) + 2\alpha_{i,12} \text{tr}(\tau^3 X_{i,\mu}) \text{tr}(X_{i,\nu} \overline{W}_i^{\mu\nu}) \\ & + \frac{1}{4}\alpha_{i,13}g \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \text{tr}(\tau^3 \overline{W}_{i,\rho\sigma}) + \frac{1}{8}\alpha_{i,14}\epsilon^{\mu\nu\rho\sigma} \text{tr}(\tau^3 \overline{W}_{i,\mu\nu}) \text{tr}(\tau^3 \overline{W}_{i,\rho\sigma}) \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{Hi} = & (\partial_\mu h) \{ \bar{\alpha}_{Hi,1} \text{tr}(\tau^3 X_i^\mu) \text{tr}(X_{i,\nu}^2) + \bar{\alpha}_{Hi,2} \text{tr}(\tau^3 X_i^\nu) \text{tr}(X_i^\mu X_{i,\nu}) + \bar{\alpha}_{Hi,3} \text{tr}(\tau^3 X_i^\nu) \text{tr}(\tau^3 X_i^\mu X_{i,\nu}) \\
 & + \bar{\alpha}_{Hi,4} \text{tr}(\tau^3 X_i^\mu) [\text{tr}(\tau^3 X_{i,\nu})]^2 + i \bar{\alpha}_{Hi,5} \text{tr}(\tau^3 X_{i,\nu}) \text{tr}(\tau^3 \bar{W}_i^{\mu\nu}) + ig \bar{\alpha}_{Hi,6} B^{\mu\nu} \text{tr}(\tau^3 X_{i,\nu}) \\
 & + i \bar{\alpha}_{Hi,7} \text{tr}(\tau^3 \bar{W}_i^{\mu\nu} X_{i,\nu}) + i \bar{\alpha}_{Hi,8} \text{tr}(\bar{W}_i^{\mu\nu} X_{i,\nu}) \} + (\partial_\mu h)(\partial_\nu h) [\bar{\alpha}_{Hi,9} \text{tr}(\tau^3 X_i^\mu) \text{tr}(\tau^3 X_i^\nu) \\
 & + \bar{\alpha}_{Hi,10} \text{tr}(X_i^\mu X_i^\nu)] + (\partial_\mu h)^2 \{ \bar{\alpha}_{Hi,11} [\text{tr}(\tau^3 X_{i,\nu})]^2 + \bar{\alpha}_{Hi,12} \text{tr}(X_{i,\nu}^2) \} \\
 & + \bar{\alpha}_{Hi,13} (\partial_\mu h)^2 (\partial_\nu h) \text{tr}(\tau^3 X_i^\nu) + \bar{\alpha}_{Hi,14} (\partial_\mu h)^4
 \end{aligned}$$

$$f(\hat{U}_R, \hat{U}_L, D_\mu \hat{U}_R, D_\mu \hat{U}_L) q_\alpha = \begin{cases} f(U_2, U_1, D_\mu U_2, D_\mu U_1) q_\alpha & \text{LR,LP} \\ f(1, U_1, 0, D_\mu U_1) q_\alpha & \text{HP,FP} \\ f(1, U_2, 0, D_\mu U_2) q_\alpha & \text{UN} \\ f(1, U_1, 0, D_\mu U_1) q_\alpha \delta_{\alpha\alpha_1} + f(1, U_2, 0, D_\mu U_2) q_\alpha \delta_{\alpha\alpha_2} & \text{NU} \end{cases}$$
  

$$f(\hat{U}_R, \hat{U}_L, D_\mu \hat{U}_R, D_\mu \hat{U}_L) l_\alpha = \begin{cases} f(U_2, U_1, D_\mu U_2, D_\mu U_1) l_\alpha & \text{LR} \\ f(1, U_1, 0, D_\mu U_1) l_\alpha & \text{LP} \\ f(U_2, U_1, D_\mu U_2, D_\mu U_1) l_\alpha & \text{HP} \\ f(1, U_1, 0, D_\mu U_1) l_\alpha & \text{FP,UN} \\ f(1, U_1, 0, D_\mu U_1) l_\alpha \delta_{\alpha\alpha_1} + f(1, U_2, 0, D_\mu U_2) l_\alpha \delta_{\alpha\alpha_2} & \text{NU} \end{cases}$$

S-Z.Wang, F-J.Ge, Q.Wang, Phys. Lett. B662, 375(2008)

$$\mathcal{L}_{Y,\text{lepton}} = \bar{l}_{\alpha L}^I [\hat{U}_L (y^{\alpha\beta} + y_3^{\alpha\beta} \tau^3) \hat{U}_R^\dagger] l_{\beta R}^I + \frac{1}{2} [h_L^{\alpha\beta} \bar{l}_{\alpha L}^{Ic} \hat{U}_L^* (1 + \tau^3) \hat{U}_L^\dagger l_{\beta L}^I + (L \rightarrow R)] + \text{h.c.}$$

$$\mathcal{L}_{Y,\text{quark}} = \bar{q}_{\alpha L}^I [\hat{U}_L (\tau^u y_u^{\alpha\beta} + \tau^d y_d^{\alpha\beta}) \hat{U}_R^\dagger] q_{\beta R}^I + \text{h.c.}$$

$$\begin{aligned} \mathcal{L}_{f-4} = i \sum_{\alpha} \left\{ & \bar{q}_{\alpha L}^I \not{D} q_{\alpha L}^I + \delta_{L,1,\alpha} \bar{q}_{\alpha L}^I \hat{U}_L (\not{D} \hat{U}_L)^\dagger q_{\alpha L}^I + \delta_{L,2,\alpha} \bar{q}_{\alpha R}^I \hat{U}_R \hat{U}_L^\dagger (\not{D} \hat{U}_L) \hat{U}_R^\dagger q_{\alpha R}^I \\ & + \delta_{L,3,\alpha} \bar{q}_{\alpha L}^I [(\not{D} \hat{U}_L) \tau^3 \hat{U}_L^\dagger - \hat{U}_L \tau^3 (\not{D} \hat{U}_L)^\dagger] q_{\alpha L}^I + \delta_{L,4,\alpha} \bar{q}_{\alpha L}^I \hat{U}_L \tau^3 \hat{U}_L^\dagger (\not{D} \hat{U}_L) \tau^3 \hat{U}_L^\dagger q_{\alpha L}^I \\ & + \delta_{L,5,\alpha} \bar{q}_{\alpha R}^I \hat{U}_R [\tau^3 \hat{U}_L^\dagger (\not{D} \hat{U}_L) - (\not{D} \hat{U}_L)^\dagger \hat{U}_L \tau^3] \hat{U}_R^\dagger q_{\alpha R}^I + \delta_{L,6,\alpha} \bar{q}_{\alpha R}^I \hat{U}_R \tau^3 \hat{U}_L^\dagger (\not{D} \hat{U}_L) \tau^3 \hat{U}_R^\dagger q_{\alpha R}^I \\ & + \delta_{L,7,\alpha} [\bar{q}_{\alpha L}^I \hat{U}_L \tau^3 \hat{U}_L^\dagger \not{D} q_{\alpha L}^I - (\bar{q}_{\alpha L}^I \not{D}^\dagger) \hat{U}_L \tau^3 \hat{U}_L^\dagger q_{\alpha L}^I] \right\} + q^I \rightarrow l^I, \delta \rightarrow \delta^l + L \leftrightarrow R \end{aligned}$$

$$D_\mu q_\alpha = \begin{cases} (\partial_\mu + ig_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + ig_2 \frac{\tau^a}{2} W_{2,\mu}^a P_R + \frac{i}{6} g B_\mu) q_\alpha & \mathbf{LR}, \mathbf{LP} \\ (\partial_\mu + ig_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + ig \frac{\tau^3}{2} B_\mu P_R + \frac{i}{6} g B_\mu) q_\alpha & \mathbf{HP}, \mathbf{FP} \\ (\partial_\mu + ig_2 \frac{\tau^a}{2} W_{2,\mu}^a P_L + ig \frac{\tau^3}{2} B_\mu P_R + \frac{i}{6} g B_\mu) q_\alpha & \mathbf{UN} \\ (\partial_\mu + i\delta_{\alpha\alpha_1} g_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + i\delta_{\alpha\alpha_2} g_2 \frac{\tau^a}{2} W_{2,\mu}^a P_L + ig \frac{\tau^3}{2} B_\mu P_R + \frac{i}{6} g B_\mu) q_\alpha & \mathbf{NU} \end{cases}$$

$$D_\mu l_\alpha = \begin{cases} (\partial_\mu + ig_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + ig_2 \frac{\tau^a}{2} W_{2,\mu}^a P_R - \frac{i}{2} g B_\mu) l_\alpha & \mathbf{LR}, \mathbf{HP} \\ (\partial_\mu + ig_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + ig \frac{\tau^3}{2} B_\mu P_R - \frac{i}{2} g B_\mu) l_\alpha & \mathbf{LP}, \mathbf{FP}, \mathbf{UN} \\ (\partial_\mu + i\delta_{\alpha\alpha_1} g_1 \frac{\tau^a}{2} W_{1,\mu}^a P_L + i\delta_{\alpha\alpha_2} g_2 \frac{\tau^a}{2} W_{2,\mu}^a P_L + ig \frac{\tau^3}{2} B_\mu P_R - \frac{i}{2} g B_\mu) l_\alpha & \mathbf{NU} \end{cases}$$

# W – W'混合

$$\tan 2\zeta = \frac{2\kappa x}{1-x^2} \quad \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \zeta & -\sin \zeta \\ \sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} W^\pm \\ W'^\pm \end{pmatrix} \quad x = \frac{f_1 g_1}{f_2 g_2}$$

$$\frac{M_W^2}{M_{W'}^2} = \frac{1+x^2 - \sqrt{(1-x^2)^2 + 4\kappa^2 x^2}}{1+x^2 + \sqrt{(1-x^2)^2 + 4\kappa^2 x^2}}$$

$$\kappa \sim 10^{-2}, \quad x \sim 10^{-1} \quad \Rightarrow \quad \zeta \sim 10^{-3}, \quad M_{W'} \sim 10 M_W$$

$$\begin{aligned} \mathcal{L}_{\text{EWCL}}^{\text{boson}} = & -V(h) + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4}f_1^2 \text{tr}(X_{1,\mu} X_1^\mu) - \frac{1}{4}f_2^2 \text{tr}(X_{2,\mu} X_2^\mu) + \frac{1}{2}\kappa f_1 f_2 \text{tr}(X_1^\mu X_2^\mu) \\ & + \frac{1}{4}\beta_{1,1} f_1^2 [\text{tr}(\tau^3 X_{1,\mu})]^2 + \frac{1}{4}\beta_{2,1} f_2^2 [\text{tr}(\tau^3 X_{2,\mu})]^2 + \frac{1}{2}\tilde{\beta}_1 f_1 f_2 [\text{tr}(\tau^3 X_{1,\mu})][\text{tr}(\tau^3 X_2^\mu)] \\ & + O(p^4) \end{aligned}$$

$$\overline{W}_{i,\mu\nu} \equiv U_i^\dagger g_i W_{i,\mu\nu} U_i \quad X_i^\mu \equiv U_i^\dagger (D^\mu U_i) \quad D_\mu U_i = \partial_\mu U_i + i g_i \frac{\tau^a}{2} W_{i,\mu}^a U_i - i g U_i \frac{\tau_3}{2} B_\mu$$



# 轻子混合

$$\mathcal{L}_{Y,\text{lepton}}|_{\text{Unitary gauge}} = \mathcal{L}_{Me} + \mathcal{L}_{M\nu} \quad \mathcal{L}_{Me} = \overline{e^{-I}}_{\alpha L} (y^{\alpha\beta} - y_3^{\alpha\beta}) e_{\beta R}^{-I} + \overline{e^{-I}}_{\alpha R} (y^{\dagger\alpha\beta} - y_3^{\dagger\alpha\beta}) e_{\beta L}^{-I}$$

$$e_{L,R}^- = \tilde{V}_{L,R}^e e_{L,R}^{-I} \quad \mathcal{L}_{M\nu} = \overline{e^-}_L \mathbf{M}^{e\dagger} e_R^- + \overline{e^-}_R \mathbf{M}^e e_L^- \quad \mathbf{M}^e = \tilde{V}_L^e (y - y_3) \tilde{V}_R^{e\dagger}$$

$$\mathcal{L}_{M\nu} = \overline{\nu^I}_{\alpha L} (y^{\alpha\beta} + y_3^{\alpha\beta}) \nu_{\beta R}^I + h_L^{\alpha\beta} \overline{\nu_{\alpha L}^{Ic}} \nu_{\beta L}^I + h_R^{\alpha\beta} \overline{\nu_{\alpha R}^{Ic}} \nu_{\beta R}^I + \text{h.c.}$$

$$= \frac{1}{2} \begin{pmatrix} \overline{\nu_L^I} & \overline{\nu_R^{Ic}} \end{pmatrix} \begin{pmatrix} 2h_L & y + y_3 \\ (y + y_3)^T & 2h_R \end{pmatrix} \begin{pmatrix} \nu_L^{Ic} \\ \nu_R^I \end{pmatrix} + \text{h.c.}$$

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} 2h_L & y + y_3 \\ (y + y_3)^T & 2h_R \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \hat{M}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix}$$

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{8}(y + y_3)h_R^{-2}(y + y_3)^T & \frac{1}{2}(y + y_3)h_R^{-1} \\ -\frac{1}{2}h_R^{-1}(y + y_3)^T & 1 - \frac{1}{8}h_R^{-1}(y + y_3)^T(y + y_3)h_R^{-1} \end{pmatrix}$$

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} 2h_L & y + y_3 \\ (y + y_3)^T & 2h_R \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^*$$

$$= \begin{pmatrix} 2h_L - \frac{1}{2}(y + y_3)h_R^{-1}(y + y_3)^T + O(h_R^{-2}) & O(h_R^{-1}) \\ O(h_R^{-1}) & 2h_R + O(h_R^{-1}) \end{pmatrix}$$



# 夸克混合

$$\mathcal{L}_{Y, \text{quark}}|_{\text{Unitary gauge}} = \bar{q}_{\alpha L}^I (\tau^u y_u^{\alpha\beta} + \tau^d y_d^{\alpha\beta}) q_{\beta R}^I + \text{h.c.}$$

$$u_{L,R} = V_{L,R}^u u_{L,R}^I \quad d_{L,R} = V_{L,R}^d u_{L,R}^I \quad V_L^u y_u^0 V_R^{u\dagger} = M_{diag}^u \quad V_L^d y_d^0 V_R^{d\dagger} = M_{diag}^d$$

$$q_{\alpha L,R} = \begin{pmatrix} u_{\alpha L,R} \\ d_{\alpha L,R} \end{pmatrix} = [(V_{L,R}^u)_{\alpha\beta} \tau^u + (V_{L,R}^d)_{\alpha\beta} \tau^d] \begin{pmatrix} u_{\beta L,R}^I \\ d_{\beta L,R}^I \end{pmatrix}$$

$$V_L^{\text{CKM}} = \begin{pmatrix} V_L^{ud} & V_L^{us} & V_L^{ub} \\ V_L^{cd} & V_L^{cs} & V_L^{cb} \\ V_L^{td} & V_L^{ts} & V_L^{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$V_R^{\text{CKM}} = \begin{pmatrix} \bar{V}_R^{ud} e^{2i\alpha_1} & \bar{V}_R^{us} e^{i(\alpha_1+\alpha_2+\beta_1)} & \bar{V}_R^{ub} e^{i(\alpha_1+\alpha_3+\beta_1+\beta_2)} \\ \bar{V}_R^{cd} e^{i(\alpha_1+\alpha_2-\beta_1)} & \bar{V}_R^{cs} e^{2i\alpha_2} & \bar{V}_R^{cb} e^{i(\alpha_2+\alpha_3+\beta_2)} \\ \bar{V}_R^{td} e^{i(\alpha_1+\alpha_3-\beta_1-\beta_2)} & \bar{V}_R^{ts} e^{i(\alpha_2+\alpha_3-\beta_2)} & \bar{V}_R^{tb} e^{2i\alpha_3} \end{pmatrix}$$

$$\begin{pmatrix} \bar{V}_R^{ud} & \bar{V}_R^{us} & \bar{V}_R^{ub} \\ \bar{V}_R^{cd} & \bar{V}_R^{cs} & \bar{V}_R^{cb} \\ \bar{V}_R^{td} & \bar{V}_R^{ts} & \bar{V}_R^{tb} \end{pmatrix} = \begin{pmatrix} \bar{c}_{12}\bar{c}_{13} & \bar{s}_{12}\bar{c}_{13} & \bar{s}_{13}e^{-i\bar{\delta}} \\ -\bar{s}_{12}\bar{c}_{23} - \bar{c}_{12}\bar{s}_{23}\bar{s}_{13}e^{i\bar{\delta}} & \bar{c}_{12}\bar{c}_{23} - \bar{s}_{12}\bar{s}_{23}\bar{s}_{13}e^{i\bar{\delta}} & \bar{s}_{23}\bar{c}_{13} \\ \bar{s}_{12}\bar{s}_{23} - \bar{c}_{12}\bar{c}_{23}\bar{s}_{13}e^{i\bar{\delta}} & -\bar{c}_{12}\bar{s}_{23} - \bar{s}_{12}\bar{c}_{23}\bar{s}_{13}e^{i\bar{\delta}} & \bar{c}_{23}\bar{c}_{13} \end{pmatrix}$$

**pseudo-manifest left-right symmetric:**  $\bar{V}_R^{\alpha\beta} = (V_L^{\alpha\beta})^*$

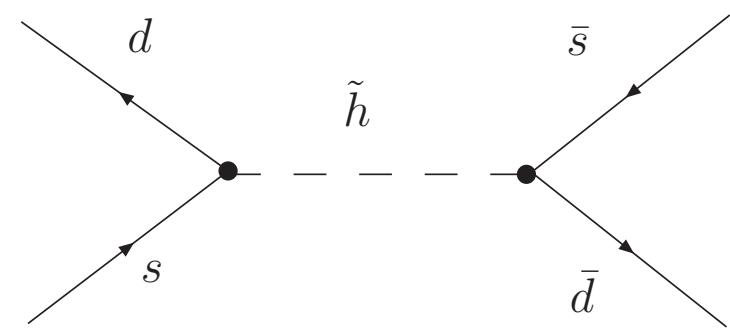
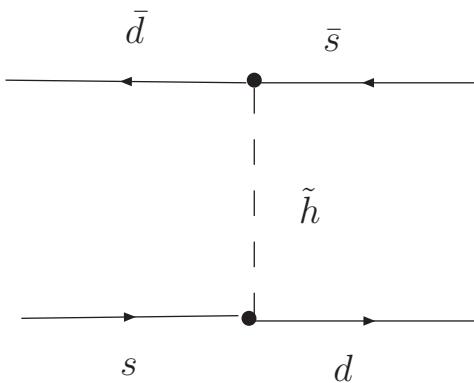
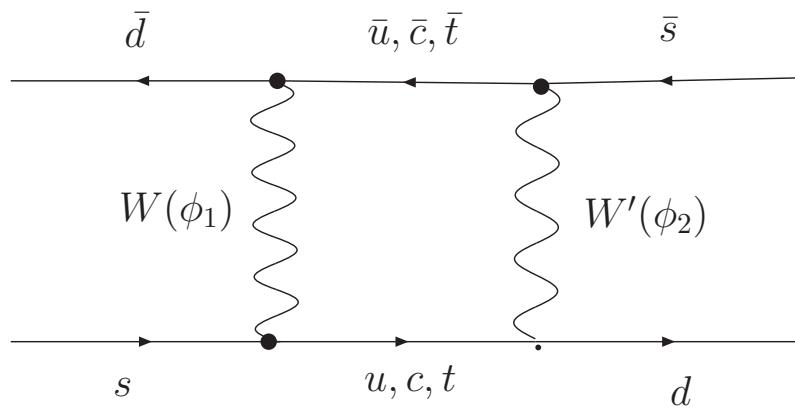
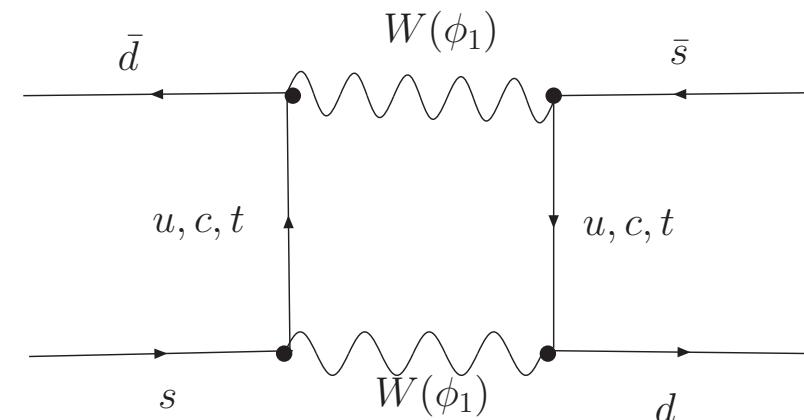
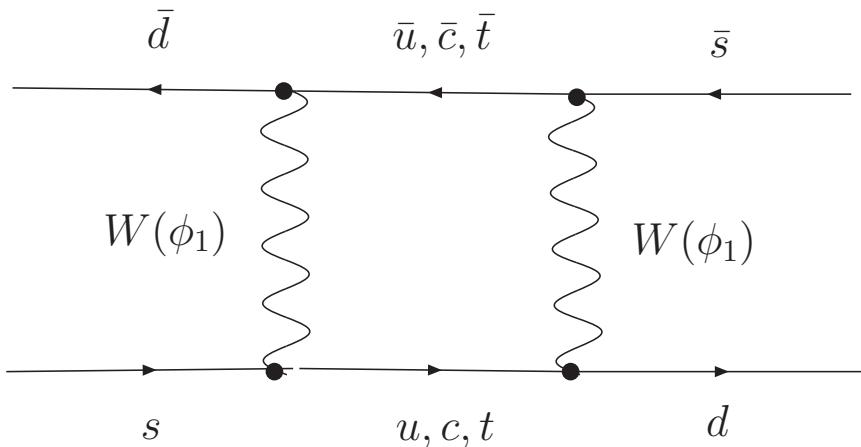
$$\Delta m_K, |\epsilon_K|, \Delta m_{B_d}, \Delta m_{B_s}$$

$$\Delta m_K = \Delta m_K^{WW} + \Delta m_K^{WW'} + \Delta m_K^h$$

$$|\epsilon_K| = |\epsilon_K|^{WW} + |\epsilon_K|^{WW'} + |\epsilon_K|^h$$

$$\Delta m_{B_d} = \Delta m_{B_d}^{WW} + \Delta m_{B_d}^{WW'} + \Delta m_{B_d}^h$$

$$\Delta m_{B_s} = \Delta m_{B_s}^{WW} + \Delta m_{B_s}^{WW'} + \Delta m_{B_s}^h$$



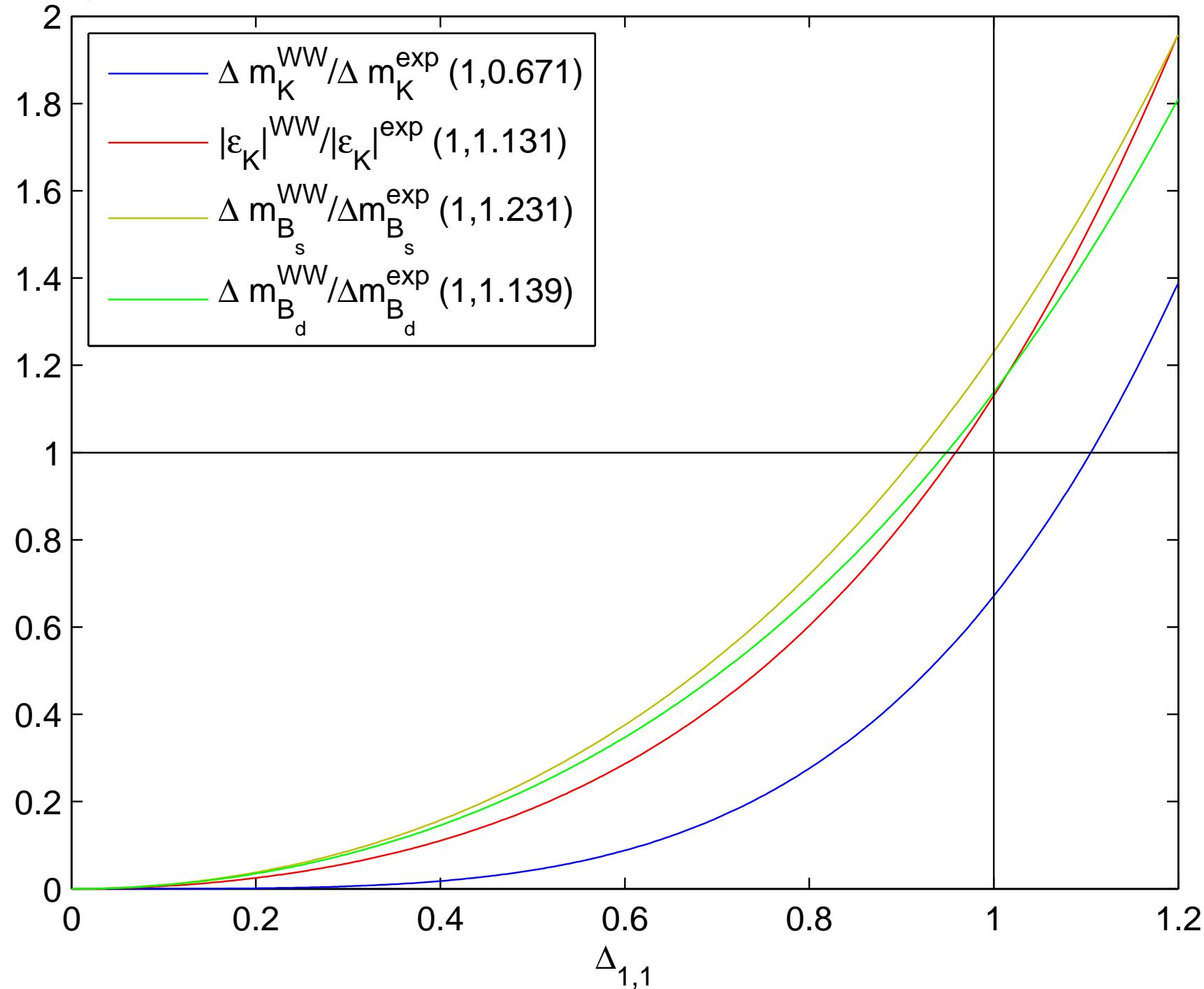
$$\Delta m_K, |\epsilon_K|, \Delta m_{B_d}, \Delta m_{B_s}$$

$$U_{1,2} = \exp\left(\frac{ig_{1,2}}{\sqrt{2}M_{1,2}}\phi_{1,2}\right) \quad \phi_{1,2} = \begin{pmatrix} \frac{\phi_{1,2}^0}{\sqrt{2}} & \phi_{1,2}^+ \\ \phi_{1,2}^- & -\frac{\phi_{1,2}^0}{\sqrt{2}} \end{pmatrix} \quad M_{1,2} = \frac{1}{2}f_{1,2}g_{1,2}$$

$$\begin{aligned} \mathcal{L}_{f=4} \Big|_{\text{Unitary gauge}} &= \sum_{\alpha} \bar{q}_{\alpha}^I [i\cancel{D} - (\Delta_{1,1,\alpha} g_1 \frac{\tau^{a'}}{2} W_1^{a'} + \Delta_{1,2,\alpha} \frac{\tau^{a'}}{2} g_2 W_2^{a'} + \Delta_{1,1,\alpha}^3 g_1 W_1^3 \\ &\quad + \Delta_{1,2,\alpha}^3 g_2 W_2^3 + \Delta_{1,\alpha} g \cancel{B}) P_L - (\Delta_{2,1,\alpha} g_1 \frac{\tau^{a'}}{2} W_1^{a'} + \Delta_{2,2,\alpha} \frac{\tau^{a'}}{2} g_2 W_2^{a'} \\ &\quad + \Delta_{2,1,\alpha}^3 g_1 W_1^3 + \Delta_{2,2,\alpha}^3 g_2 W_2^3 + \Delta_{2,\alpha} g \cancel{B}) P_R] q_{\alpha}^I + q^I \rightarrow l^I, \delta \rightarrow \delta^l, \Delta \rightarrow \Delta^l, \end{aligned}$$

$$u_{L,R} = V_{L,R}^u u_{L,R}^I \quad d_{L,R} = V_{L,R}^d u_{L,R}^I \quad V_L^u y_u^0 V_R^{u\dagger} = M_{diag}^u \quad V_L^d y_d^0 V_R^{d\dagger} = M_{diag}^d$$

$$\Delta_{1,1,\alpha} = 1 - \delta_{L,1,\alpha} - \delta_{L,4,\alpha} \quad \text{LR,LP,HP,FP} \quad \Delta_{2,1,\alpha} = \begin{cases} 1 - \delta_{R,1,\alpha} - \delta_{R,4,\alpha} & \text{LR,LP} \\ \delta_{L,2,\alpha} - \delta_{L,6,\alpha} & \text{HP,FP} \end{cases}$$



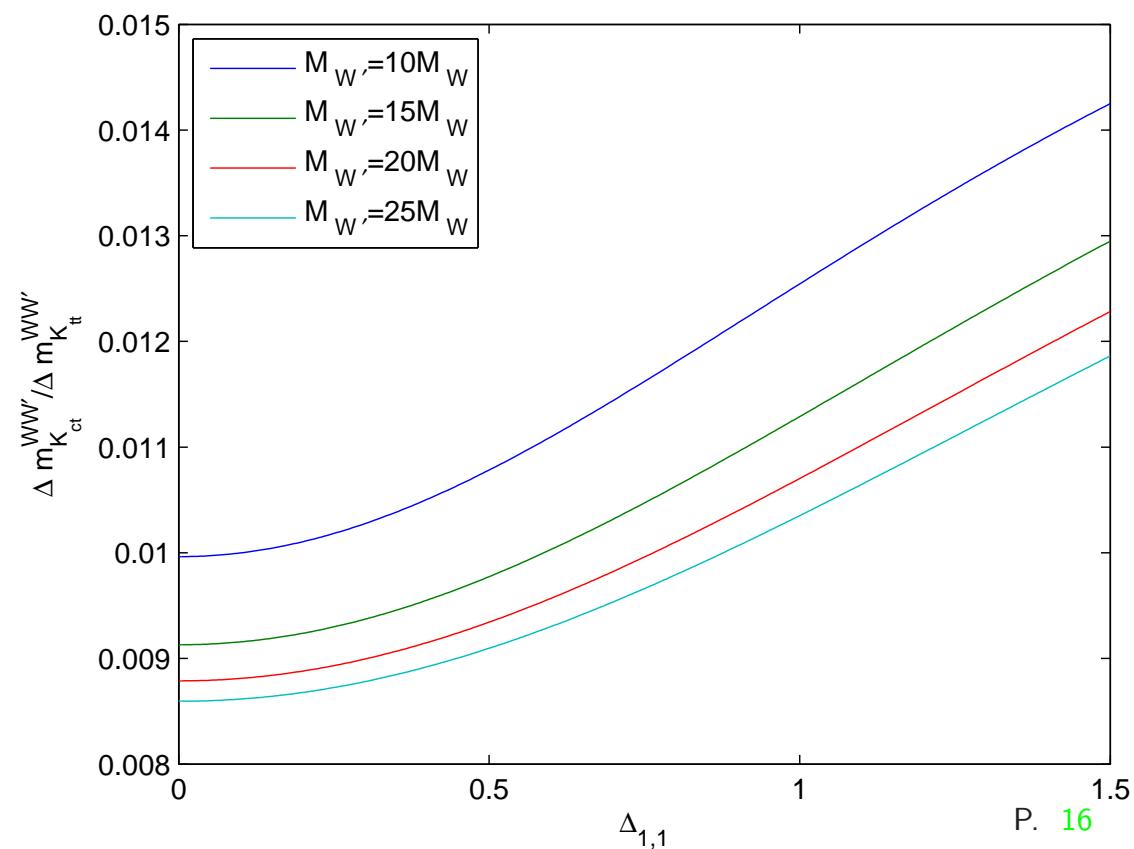
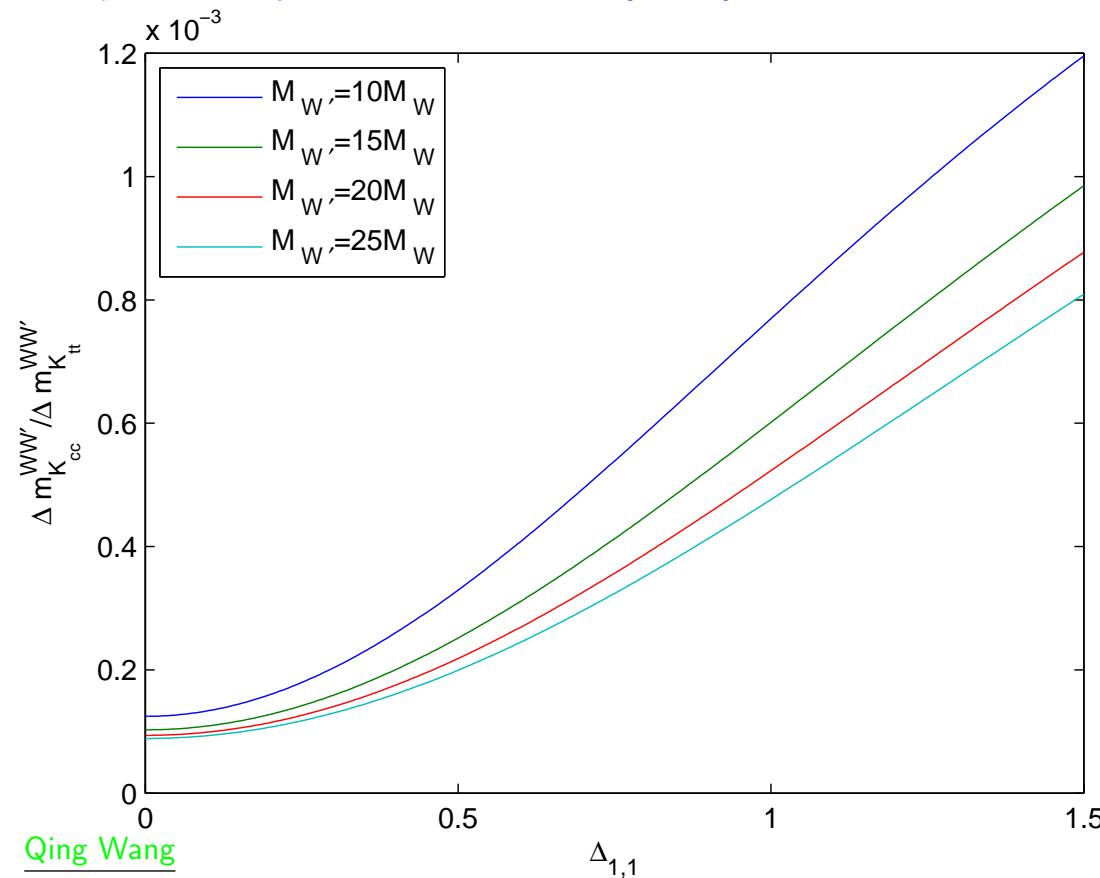


$$\Delta m_K^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{Ktt}^{WW'} \left[ \text{Re}(\lambda_t^{LR} \lambda_t^{RL}) + \text{Re}(\lambda_c^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{Kcc}^{WW'}}{\Delta m_{Ktt}^{WW'}}}_{10^{-3}} + \text{Re}(\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{Kct}^{WW'}}{\Delta m_{Ktt}^{WW'}}}_{10^{-2}} \right]$$

$$\lambda_x^{LR}(K) \lambda_x^{RL}(K) = |V_L^{xs} V_L^{xd*} \bar{V}_R^{xs} \bar{V}_R^{xd*}| e^{-i(\alpha_1 - \alpha_2 - \beta_1 - \phi_{xs} - \bar{\phi}_{xs} + \phi_{xd} + \bar{\phi}_{xd})} \quad x = c, t$$

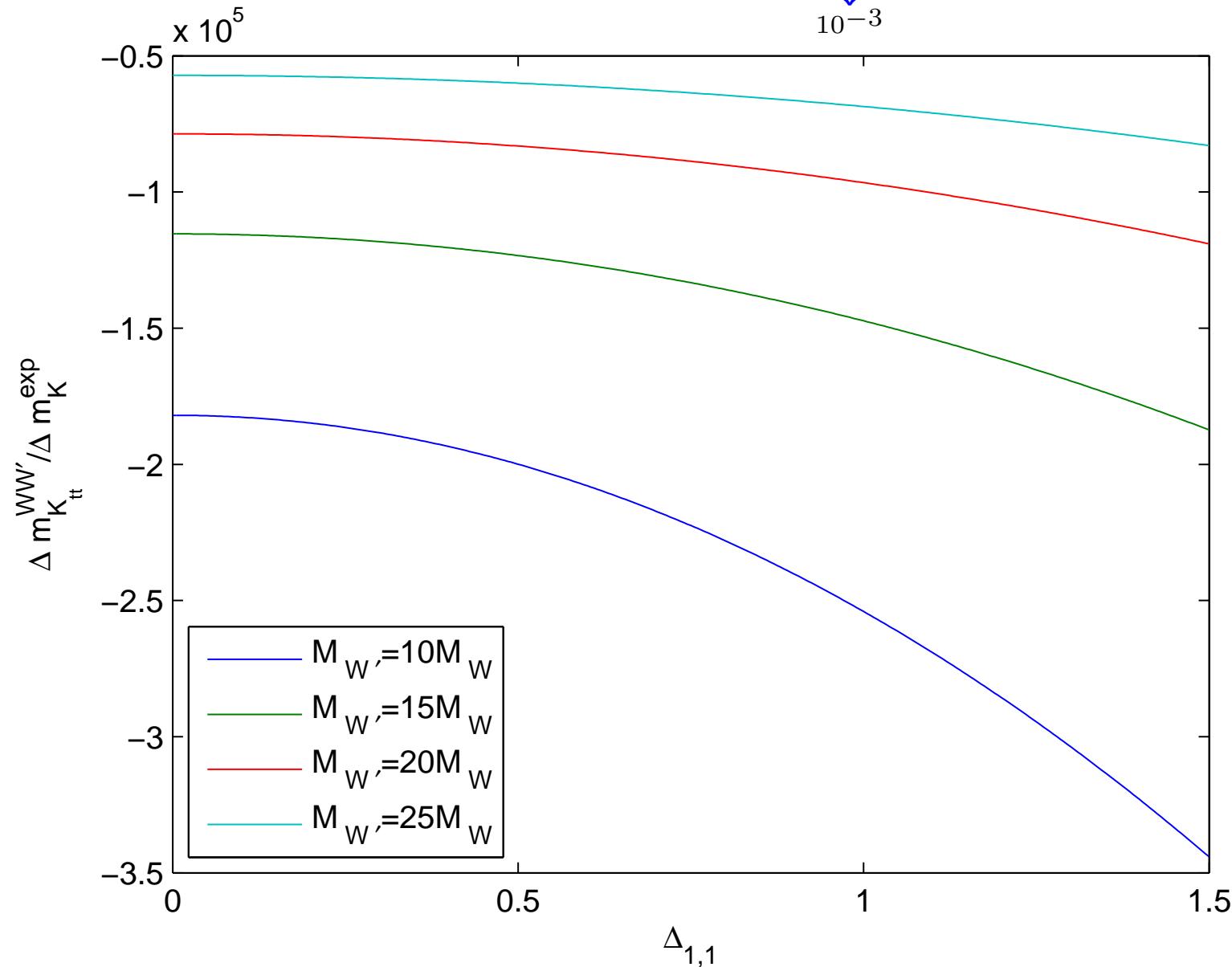
$$\lambda_c^{LR}(K) \lambda_t^{RL}(K) = |V_L^{cs} \bar{V}_R^{cd*} \bar{V}_R^{ts} V_L^{td*}| e^{-i(\alpha_1 - \alpha_3 - \beta_1 + \beta_2 - \phi_{cs} + \bar{\phi}_{cd} - \bar{\phi}_{ts} + \phi_{td})}$$

$$\lambda_t^{LR}(K) \lambda_c^{RL}(K) = |V_L^{ts} \bar{V}_R^{td*} \bar{V}_R^{cs} V_L^{cd*}| e^{-i(\alpha_1 - 2\alpha_2 + \alpha_3 - \beta_1 - \beta_2 - \phi_{ts} + \bar{\phi}_{td} - \bar{\phi}_{cs} + \phi_{cd})}$$





$$\Delta m_K^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{Ktt}^{WW'} \left[ \text{Re}(\lambda_t^{LR} \lambda_t^{RL}) + \text{Re}(\lambda_c^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{Kcc}^{WW'}}{\Delta m_{Ktt}^{WW'}}}_{10^{-3}} + \text{Re}(\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{Kct}^{WW'}}{\Delta m_{Ktt}^{WW'}}}_{10^{-2}} \right]$$



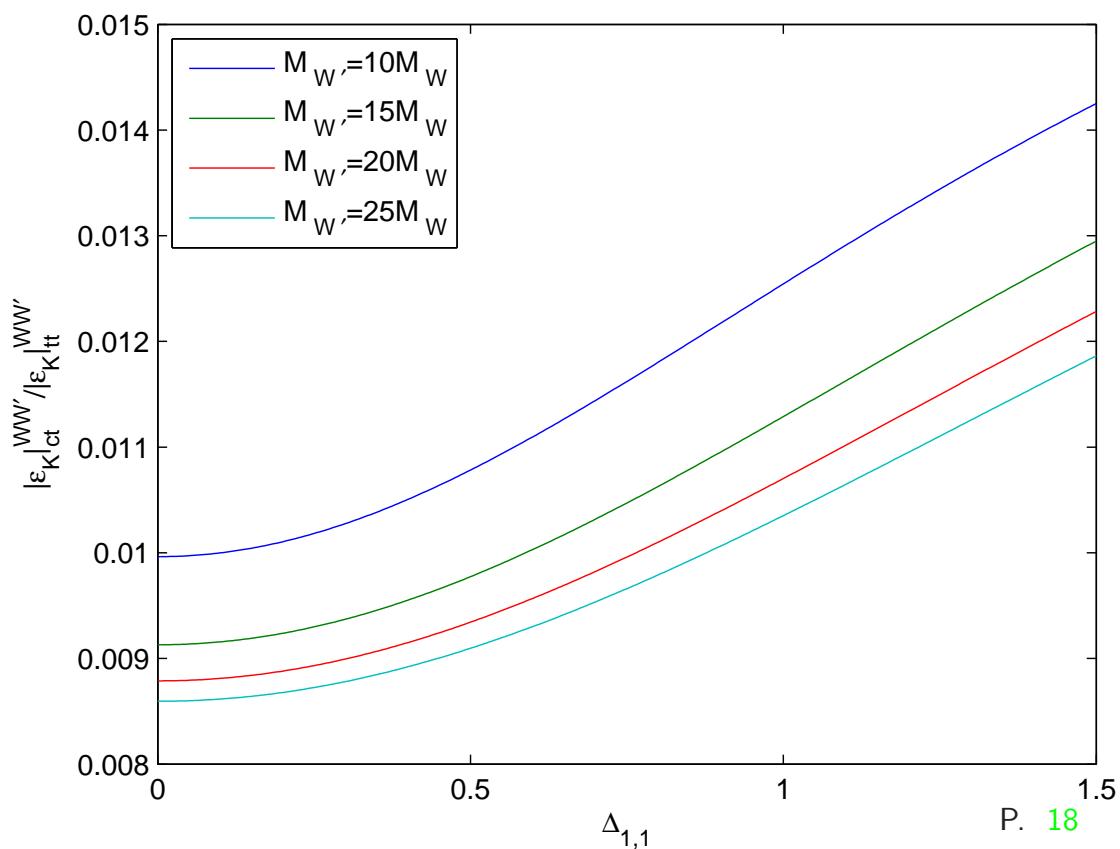
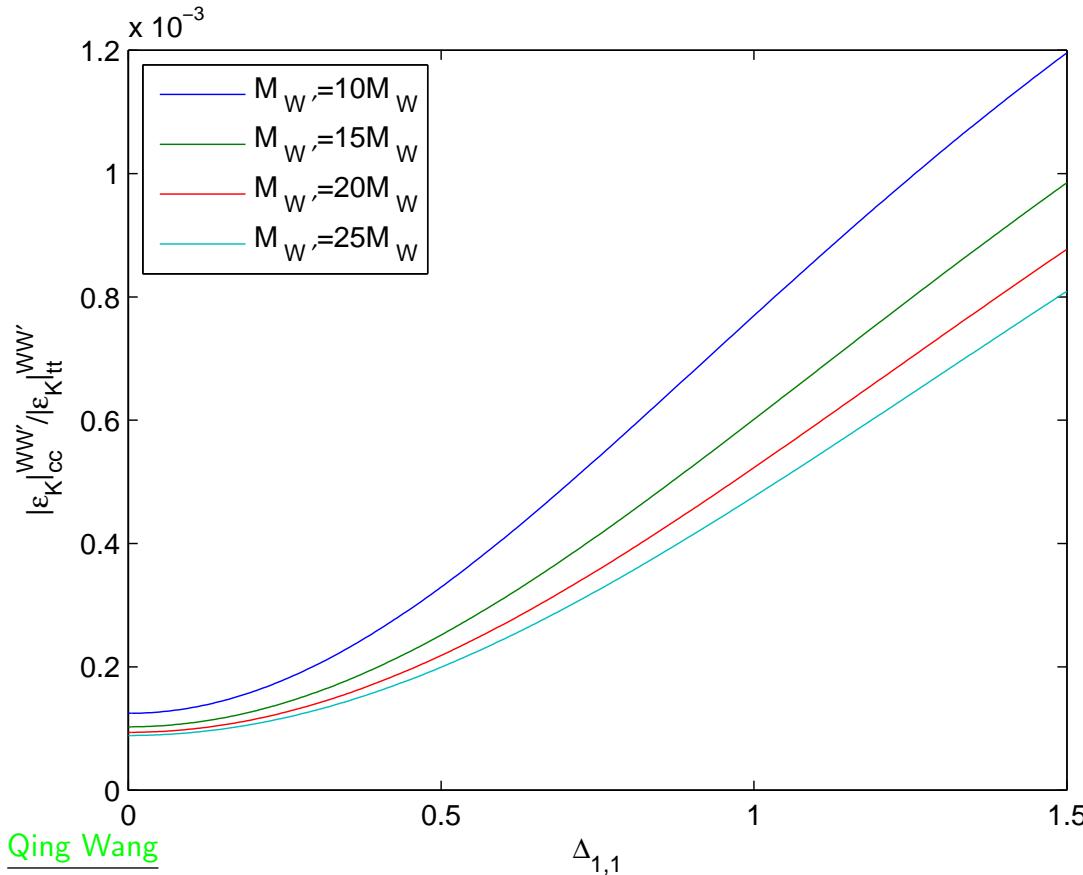


$$|\epsilon_K|^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} |\epsilon_K|_{tt}^{WW'} \left| \text{Im}(\lambda_t^{LR} \lambda_t^{RL}) + \text{Im}(\lambda_c^{LR} \lambda_c^{RL}) \underbrace{\frac{|\epsilon_K|_{cc}^{WW'}}{|\epsilon_K|_{tt}^{WW'}}}_{10^{-3}} + \text{Im}(\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{|\epsilon_K|_{ct}^{WW'}}{|\epsilon_K|_{tt}^{WW'}}}_{10^{-2}} \right|$$

$$\lambda_x^{LR}(K) \lambda_x^{RL}(K) = |V_L^{xs} V_L^{xd*} \bar{V}_R^{xs} \bar{V}_R^{xd*}| e^{-i(\alpha_1 - \alpha_2 - \beta_1 - \phi_{xs} - \bar{\phi}_{xs} + \phi_{xd} + \bar{\phi}_{xd})} \quad x = c, t$$

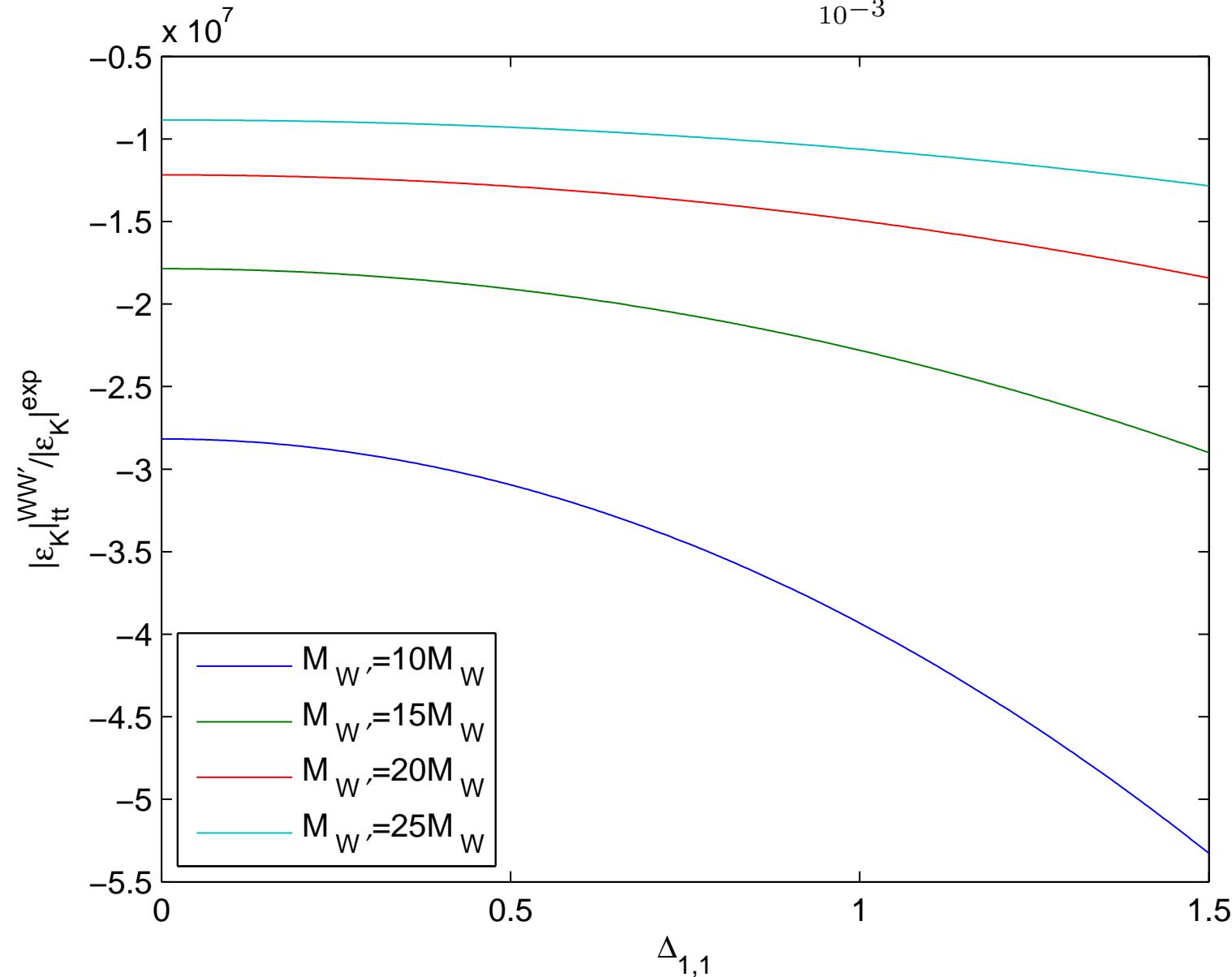
$$\lambda_c^{LR}(K) \lambda_t^{RL}(K) = |V_L^{cs} \bar{V}_R^{cd*} \bar{V}_R^{ts} V_L^{td*}| e^{-i(\alpha_1 - \alpha_3 - \beta_1 + \beta_2 - \phi_{cs} + \bar{\phi}_{cd} - \bar{\phi}_{ts} + \phi_{td})} \quad \arg(V_L^{\alpha\beta}) = \phi_{\alpha\beta}$$

$$\lambda_t^{LR}(K) \lambda_c^{RL}(K) = |V_L^{ts} \bar{V}_R^{td*} \bar{V}_R^{cs} V_L^{cd*}| e^{-i(\alpha_1 - 2\alpha_2 + \alpha_3 - \beta_1 - \beta_2 - \phi_{ts} + \bar{\phi}_{td} - \bar{\phi}_{cs} + \phi_{cd})} \quad \arg(\bar{V}_R^{\alpha\beta}) = \bar{\phi}_{\alpha\beta}$$





$$|\epsilon_K|^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} |\epsilon_K|_{tt}^{WW'} \left| \text{Im}(\lambda_t^{LR} \lambda_t^{RL}) + \text{Im}(\lambda_c^{LR} \lambda_c^{RL}) \underbrace{\frac{|\epsilon_K|_{cc}^{WW'}}{|\epsilon_K|_{tt}^{WW'}}}_{10^{-3}} + \text{Im}(\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{|\epsilon_K|_{ct}^{WW'}}{|\epsilon_K|_{tt}^{WW'}}}_{10^{-2}} \right|$$





# pseudo-manifest left-right symmetric case

$$\bar{V}_R^{\alpha\beta} = (V_L^{\alpha\beta})^* \quad \Rightarrow \quad \phi_{\alpha\beta} + \bar{\phi}_{\alpha\beta} = 0$$

$$\begin{aligned} \lambda_c^{LR}(K)\lambda_c^{RL}(K) &= |V_L^{cs}V_L^{cd}|^2 e^{-i(\alpha_1-\alpha_2-\beta_1)} \\ \lambda_t^{LR}(K)\lambda_t^{RL}(K) &= |V_L^{ts}V_L^{td}|^2 e^{-i(\alpha_1-\alpha_2-\beta_1)} \\ \lambda_c^{LR}(K)\lambda_t^{RL}(K) + \lambda_t^{LR}(K)\lambda_c^{RL}(K) &= 2|V_L^{cs}V_L^{cd}V_L^{ts}V_L^{td}|[\cos(\alpha_1 - \alpha_2 - \beta_1)\cos(\alpha_2 - \alpha_3 + \beta_2) \\ &\quad - i\sin(\alpha_1 - \alpha_2 - \beta_1)\cos(\alpha_2 - \alpha_3 + \beta_2)] \end{aligned}$$

$$\alpha_1 - \alpha_2 - \beta_1 = 0 \Rightarrow \text{Im}[\lambda_c^{LR}(K)\lambda_c^{RL}(K)] = \text{Im}[\lambda_t^{LR}(K)\lambda_t^{RL}(K)] = \text{Im}[\lambda_c^{LR}(K)\lambda_t^{RL}(K) + \lambda_t^{LR}(K)\lambda_c^{RL}(K)] = 0$$

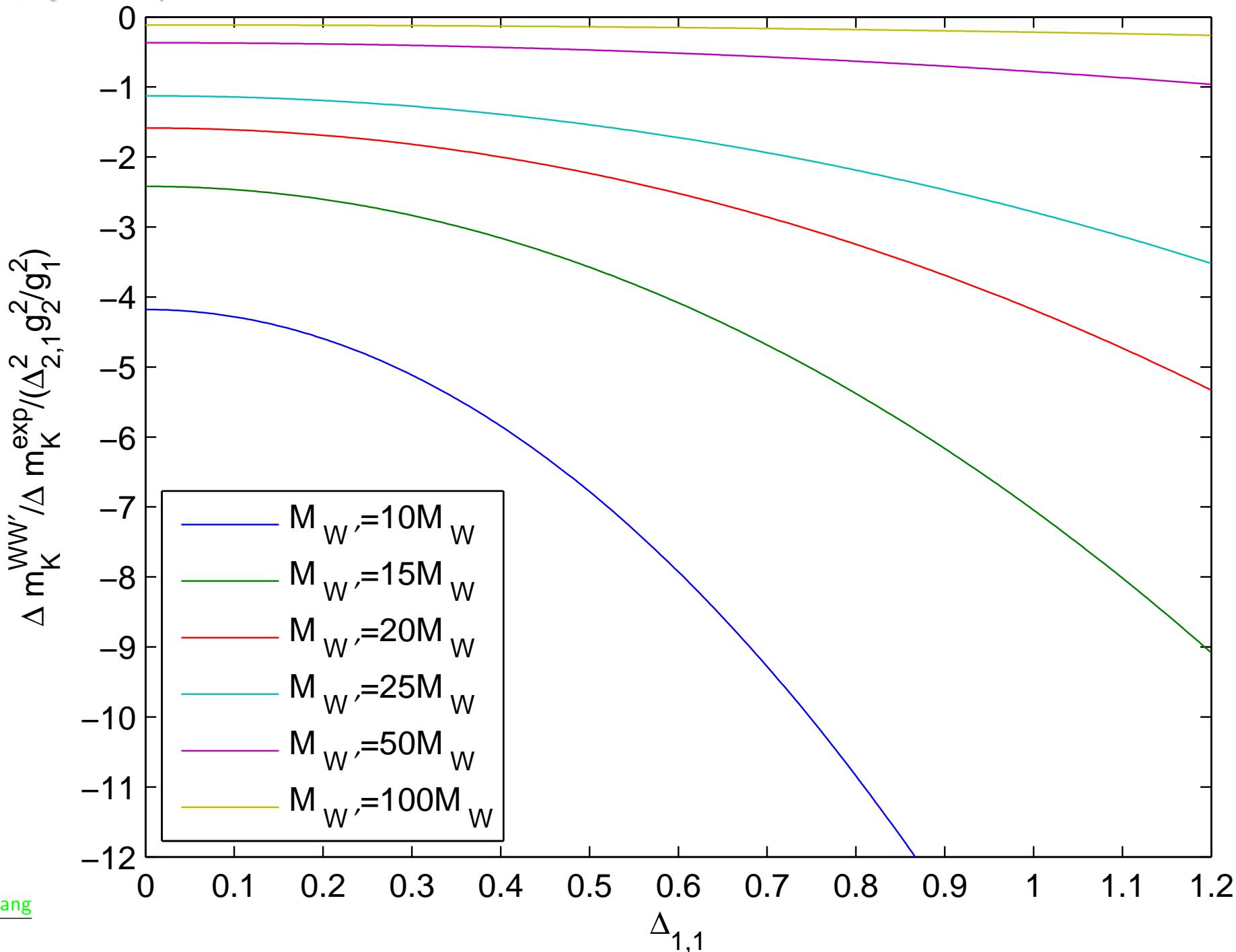
cc、tt和ct圈分别无CP破坏！

$$|\epsilon_K|^{WW'} = 0$$

$$\frac{\Delta m_K^{WW'}}{\Delta m_K^{\exp}} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \underbrace{\frac{\Delta m_{K_{tt}}^{WW'}}{\Delta m_K^{\exp}}}_{10^5} \left[ \underbrace{\text{Re}(\lambda_t^{LR}\lambda_t^{RL})}_{5.9 \times 10^{-6}} + \underbrace{\text{Re}(\lambda_c^{LR}\lambda_c^{RL})}_{0.049} \underbrace{\frac{\Delta m_{K_{cc}}^{WW'}}{\Delta m_{K_{tt}}^{WW'}}}_{10^{-3}} + \underbrace{\text{Re}(\lambda_c^{LR}\lambda_t^{RL} + \lambda_t^{LR}\lambda_c^{RL})}_{0.0011 \times \cos(\alpha_2 - \alpha_3 + \beta_2)} \underbrace{\frac{\Delta m_{K_{ct}}^{WW'}}{\Delta m_{K_{tt}}^{WW'}}}_{10^{-2}} \right]$$

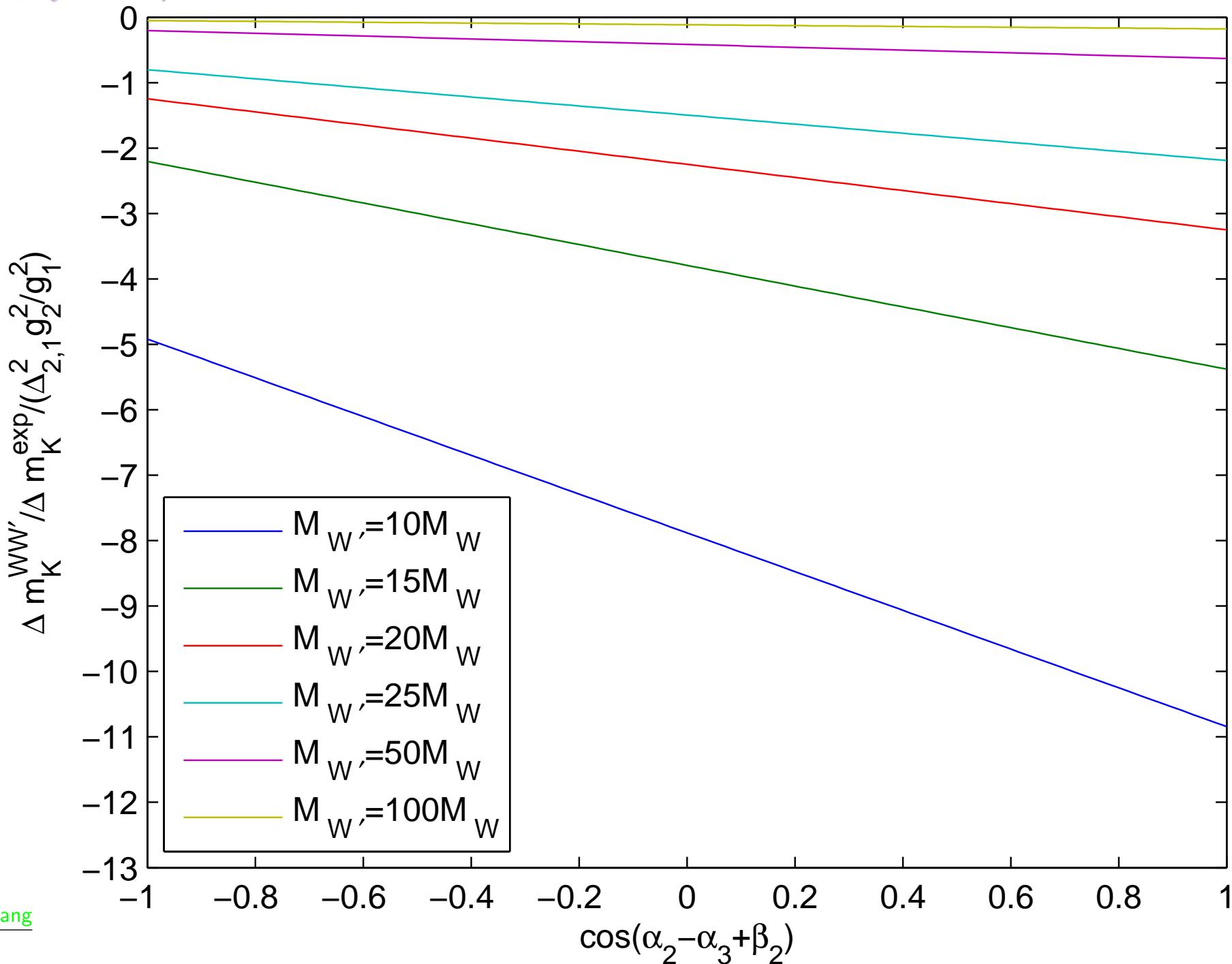


$$\cos(\alpha_2 - \alpha_3 + \beta_2) = 1$$





$\Delta_{1,1}=1$





$$\Delta m_{B_d}^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{B_d tt}^{WW'} \left| \lambda_t^{LR} \lambda_t^{RL} + \lambda_c^{LR} \lambda_c^{RL} \underbrace{\frac{\Delta m_{B_d cc}^{WW'}}{\Delta m_{B_d tt}^{WW'}}}_{10^{-3}} + (\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{B_d ct}^{WW'}}{\Delta m_{B_d tt}^{WW'}}}_{10^{-2}} \right|$$

$$\lambda_x^{LR}(B_d) \lambda_x^{RL}(B_d) = |V_L^{xb} V_L^{xd*} \bar{V}_R^{xb} \bar{V}_R^{xd*}| e^{-i(\alpha_1 - \alpha_3 - \beta_1 - \beta_2 - \phi_{xb} - \bar{\phi}_{xb} + \phi_{xd} + \bar{\phi}_{xd})}$$

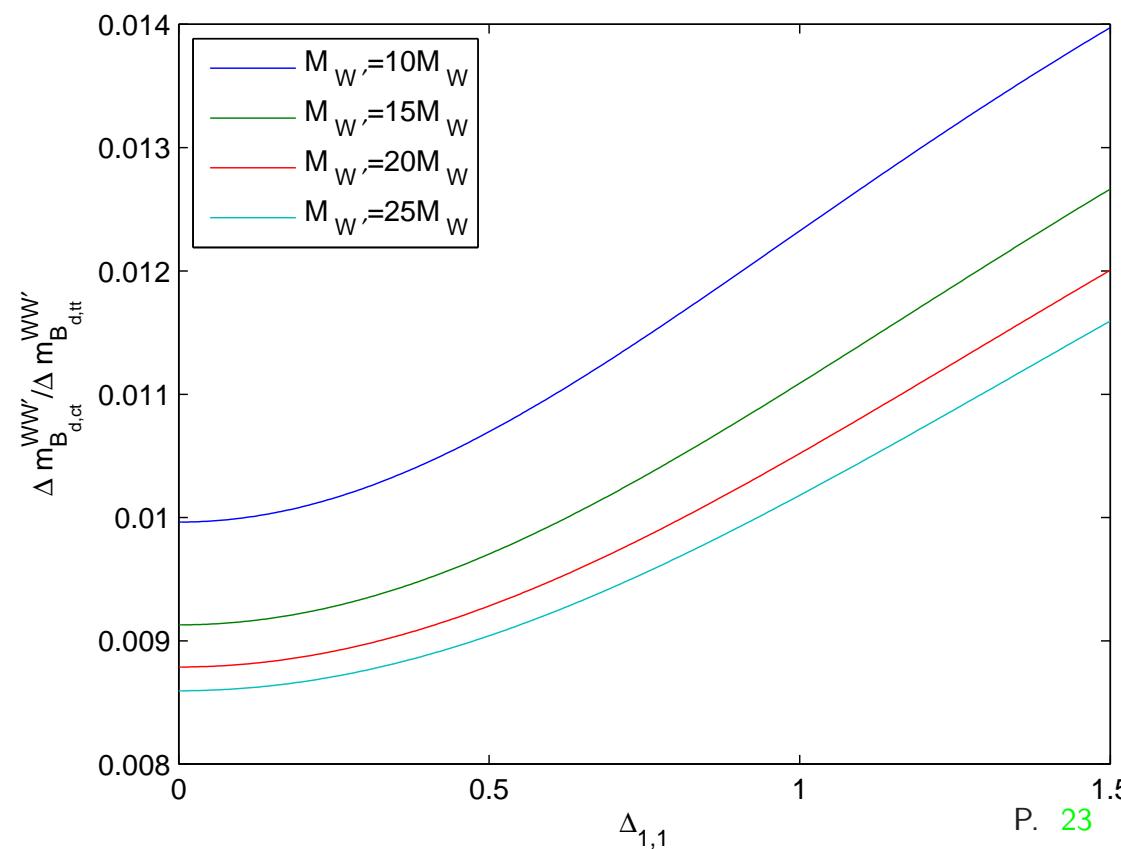
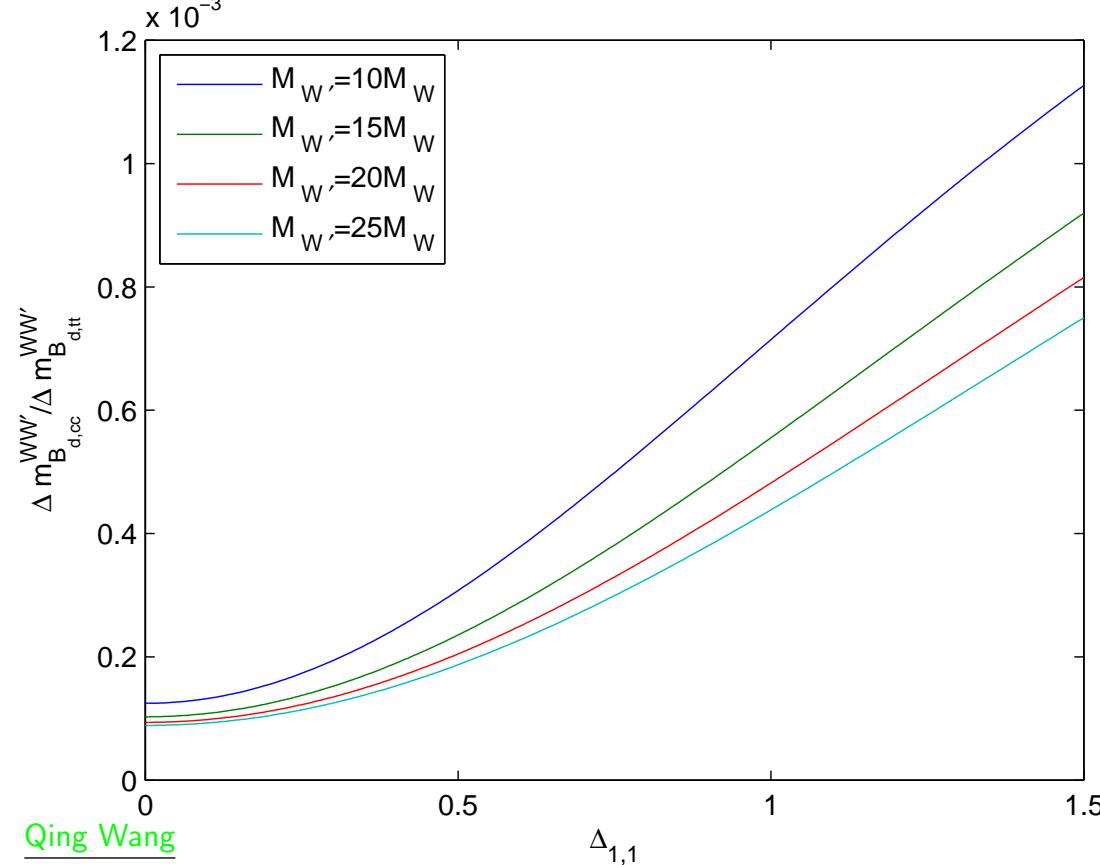
$$\lambda_c^{LR}(B_d) \lambda_t^{RL}(B_d) = |V_L^{cb} \bar{V}_R^{cd*} \bar{V}_R^{tb} V_L^{td*}| e^{-i(\alpha_1 + \alpha_2 - 2\alpha_3 - \beta_1 - \phi_{cb} + \bar{\phi}_{cd} - \bar{\phi}_{tb} + \phi_{td})}$$

$$\lambda_t^{LR}(B_d) \lambda_c^{RL}(B_d) = |V_L^{tb} \bar{V}_R^{td*} \bar{V}_R^{cb} V_L^{cd*}| e^{-i(\alpha_1 - \alpha_2 - \beta_1 - 2\beta_2 - \phi_{tb} + \bar{\phi}_{td} - \bar{\phi}_{cb} + \phi_{cd})}$$

$$x = c, t$$

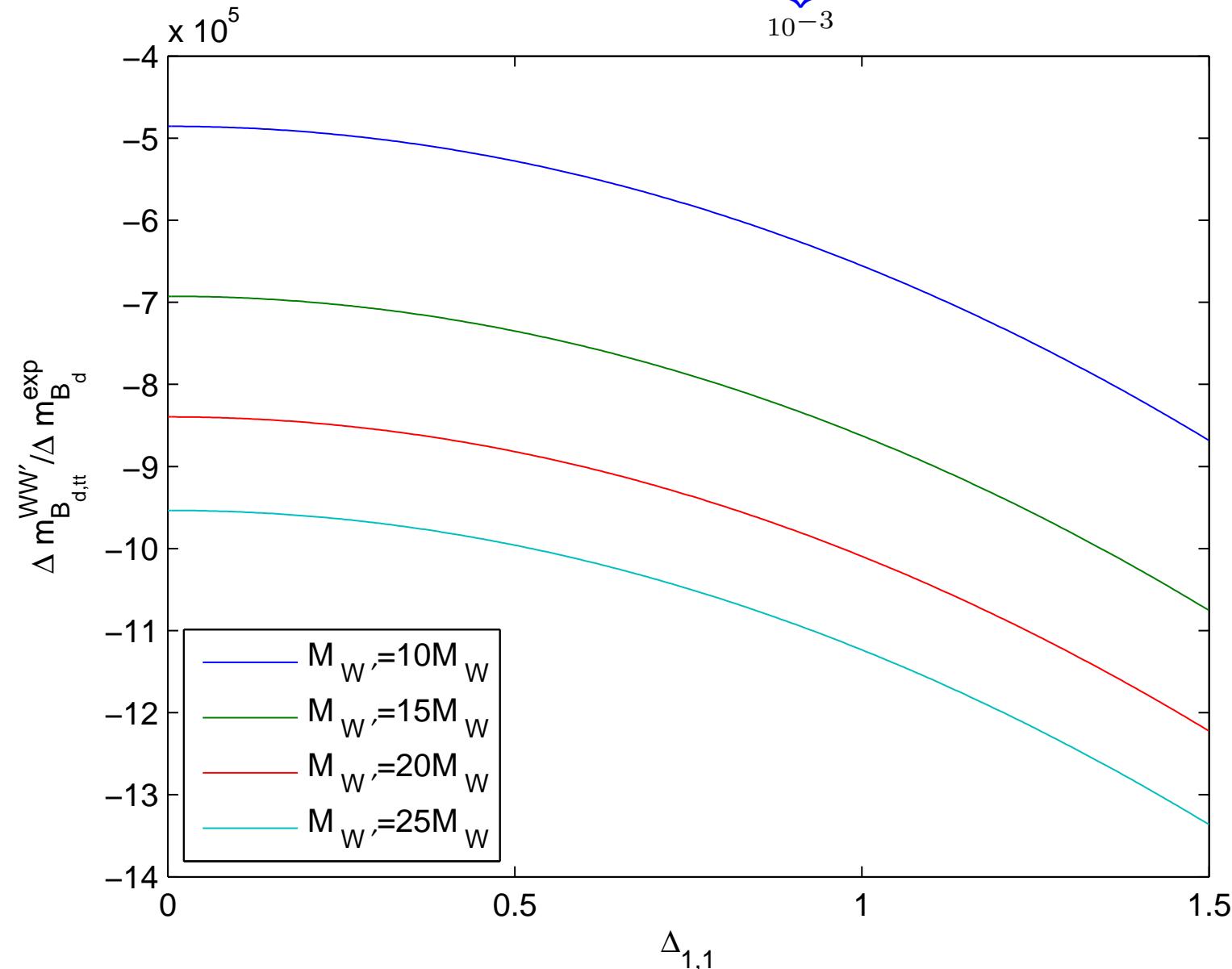
$$\arg(V_L^{\alpha\beta}) = \phi_{\alpha\beta}$$

$$\arg(\bar{V}_R^{\alpha\beta}) = \bar{\phi}_{\alpha\beta}$$





$$\Delta m_{B_d}^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{B_d tt}^{WW'} \left| \lambda_t^{LR} \lambda_t^{RL} + \lambda_c^{LR} \lambda_c^{RL} \underbrace{\frac{\Delta m_{B_d cc}^{WW'}}{\Delta m_{B_d tt}^{WW'}}}_{10^{-3}} + (\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{B_d ct}^{WW'}}{\Delta m_{B_d tt}^{WW'}}}_{10^{-2}} \right|$$





$$\Delta m_{B_s}^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{B_{s,t,t}}^{WW'} \left| \lambda_t^{LR} \lambda_t^{RL} + \lambda_c^{LR} \lambda_c^{RL} \underbrace{\frac{\Delta m_{B_{s,c,c}}^{WW'}}{\Delta m_{B_{s,t,t}}^{WW'}}}_{10^{-3}} + (\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{B_{s,c,t}}^{WW'}}{\Delta m_{B_{s,t,t}}^{WW'}}}_{10^{-2}} \right|$$

$$\lambda_x^{LR}(B_s) \lambda_x^{RL}(B_s) = |V_L^{xb} V_L^{xs*} \bar{V}_R^{xb} \bar{V}_R^{xs*}| e^{-i(\alpha_2 - \alpha_3 - \beta_2 - \phi_{xb} - \bar{\phi}_{xb} + \phi_{xs} + \bar{\phi}_{xs})}$$

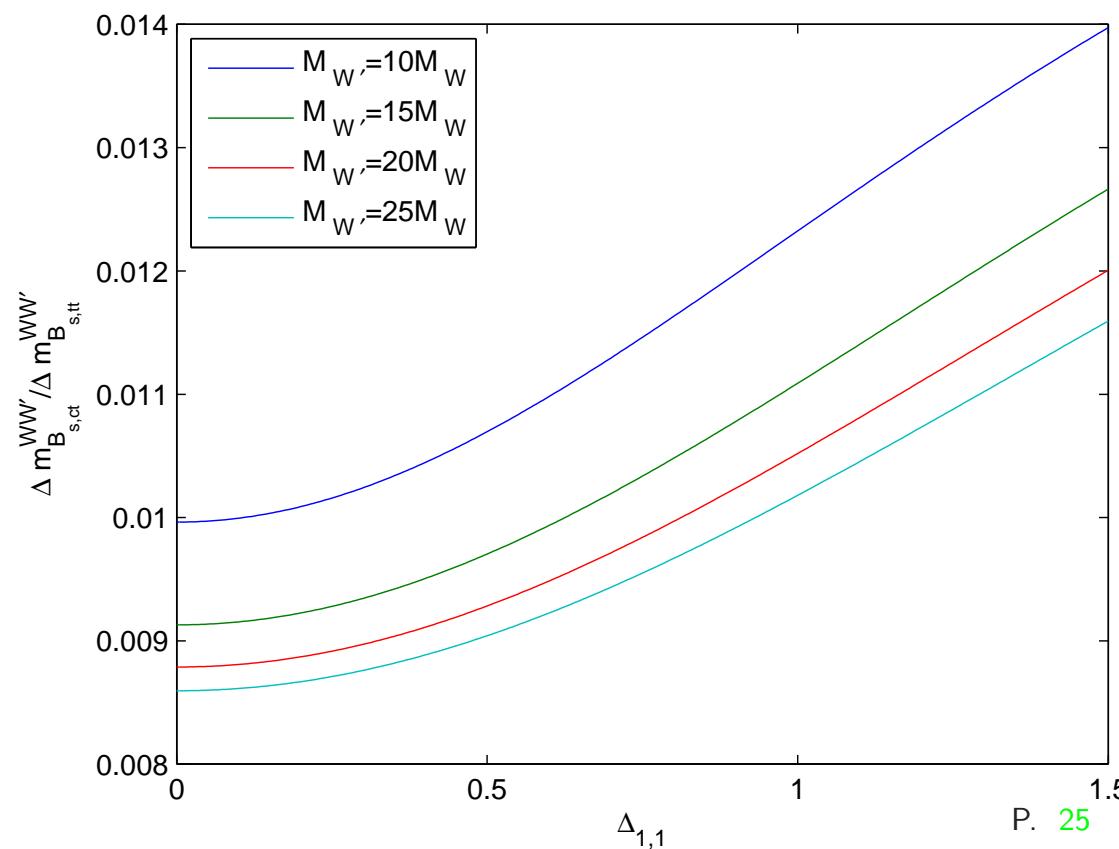
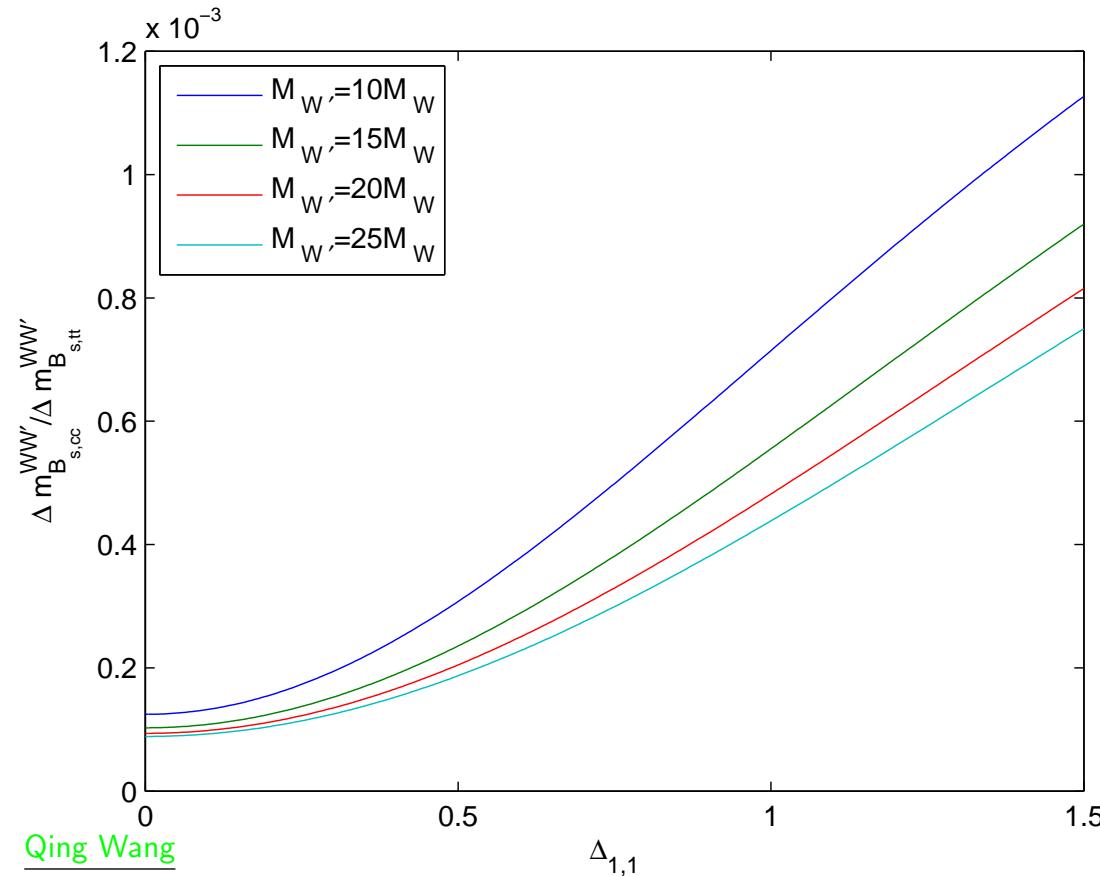
$x = c, t$

$$\lambda_c^{LR}(B_s) \lambda_t^{RL}(B_s) = |V_L^{cb} \bar{V}_R^{cs*} \bar{V}_R^{tb} V_L^{ts*}| e^{-i(2\alpha_2 - 2\alpha_3 - \phi_{cb} + \bar{\phi}_{cs} - \bar{\phi}_{tb} + \phi_{ts})}$$

$$\arg(V_L^{\alpha\beta}) = \phi_{\alpha\beta}$$

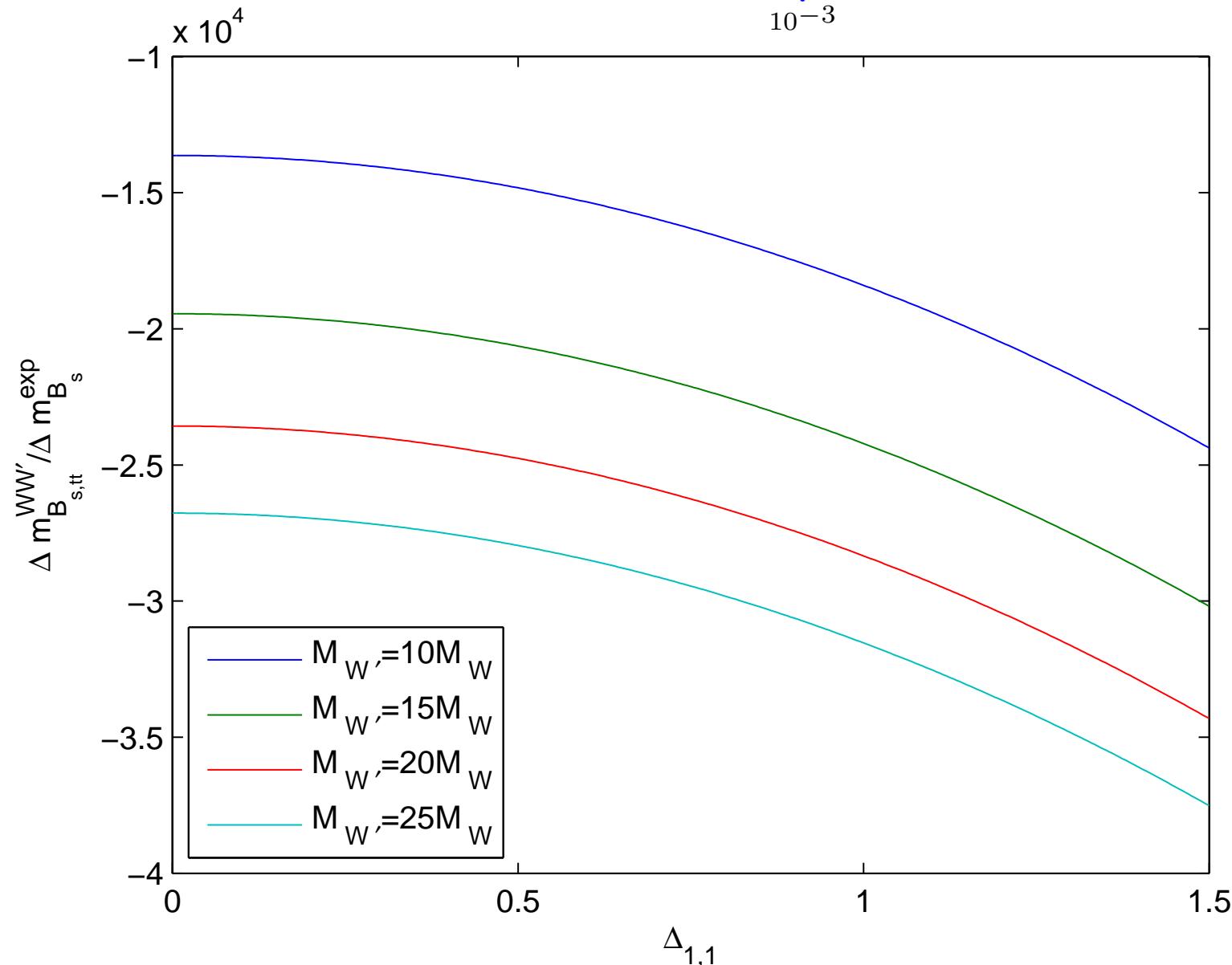
$$\lambda_t^{LR}(B_s) \lambda_c^{RL}(B_s) = |V_L^{tb} \bar{V}_R^{ts*} \bar{V}_R^{cb} V_L^{cs*}| e^{-i(-2\beta_2 - \phi_{tb} + \bar{\phi}_{ts} - \bar{\phi}_{cb} + \phi_{cs})}$$

$$\arg(\bar{V}_R^{\alpha\beta}) = \bar{\phi}_{\alpha\beta}$$





$$\Delta m_{B_s}^{WW'} = \Delta_{2,1}^2 \frac{g_2^2}{g_1^2} \Delta m_{B_s tt}^{WW'} \left| \lambda_t^{LR} \lambda_t^{RL} + \lambda_c^{LR} \lambda_c^{RL} \underbrace{\frac{\Delta m_{B_s cc}^{WW'}}{\Delta m_{B_s tt}^{WW'}}}_{10^{-3}} + (\lambda_c^{LR} \lambda_t^{RL} + \lambda_t^{LR} \lambda_c^{RL}) \underbrace{\frac{\Delta m_{B_s ct}^{WW'}}{\Delta m_{B_s tt}^{WW'}}}_{10^{-2}} \right|$$



$$\Delta m_K, |\epsilon_K|, \Delta m_{B_d}, \Delta m_{B_s}$$

$$\mathcal{L}_{Y,\text{quark}}|_{\text{Unitary gauge}} = \bar{q}_{\alpha L}^I (\tau^u y_u^{\alpha\beta} + \tau^d y_d^{\alpha\beta}) q_{\beta R}^I + \text{h.c.} \quad y_i = y_i^0 + y_i^1 \tilde{h} + \dots \quad \tilde{y}_i = V_L^i y_i^1 V_R^{i\dagger} \quad i=u,d$$

$$\Delta m_K^h \ll \Delta m_K^{\text{exp}} \Rightarrow \text{Re}[(\tilde{y}_d^{ds} + \tilde{y}_d^{\dagger ds})^2 - 11.4(\tilde{y}_d^{ds} - \tilde{y}_d^{\dagger ds})^2] \ll 6.45 \times 10^{-7} \left( \frac{m_h}{1\text{TeV}} \right)^2$$

$$|\epsilon_K|^h \ll |\epsilon_K|^{\text{exp}} \Rightarrow \text{Im}[(\tilde{y}_d^{ds} + \tilde{y}_d^{\dagger ds})^2 - 11.4(\tilde{y}_d^{ds} - \tilde{y}_d^{\dagger ds})^2] \\ \ll 0.0065 \times \text{Re}[(\tilde{y}_d^{ds} + \tilde{y}_d^{\dagger ds})^2 - 11.4(\tilde{y}_d^{ds} - \tilde{y}_d^{\dagger ds})^2]$$

$$\Delta m_{B_d}^h \ll \Delta m_{B_d}^{\text{exp}} \Rightarrow [(\tilde{y}_d^{db} + \tilde{y}_d^{\dagger db})^2 - 11.4(\tilde{y}_d^{db} - \tilde{y}_d^{\dagger db})^2] \ll 1.74 \times 10^{-6} \left( \frac{m_h}{1\text{TeV}} \right)^2$$

$$\Delta m_{B_s}^h \ll \Delta m_{B_s}^{\text{exp}} \Rightarrow [(\tilde{y}_d^{sb} + \tilde{y}_d^{\dagger sb})^2 - 11.4(\tilde{y}_d^{sb} - \tilde{y}_d^{\dagger sb})^2] \ll 6.10 \times 10^{-5} \left( \frac{m_h}{1\text{TeV}} \right)^2$$



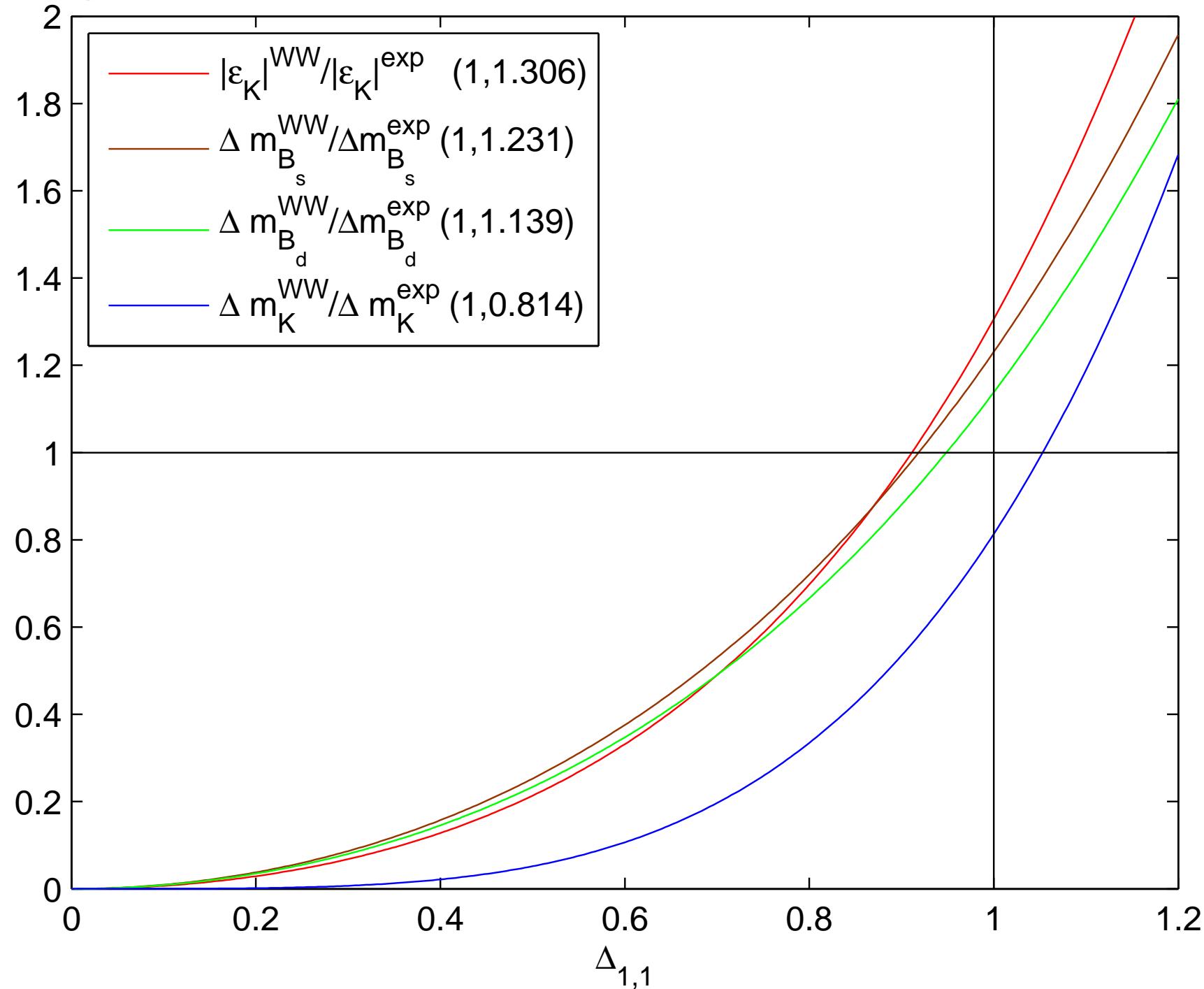
## 小结

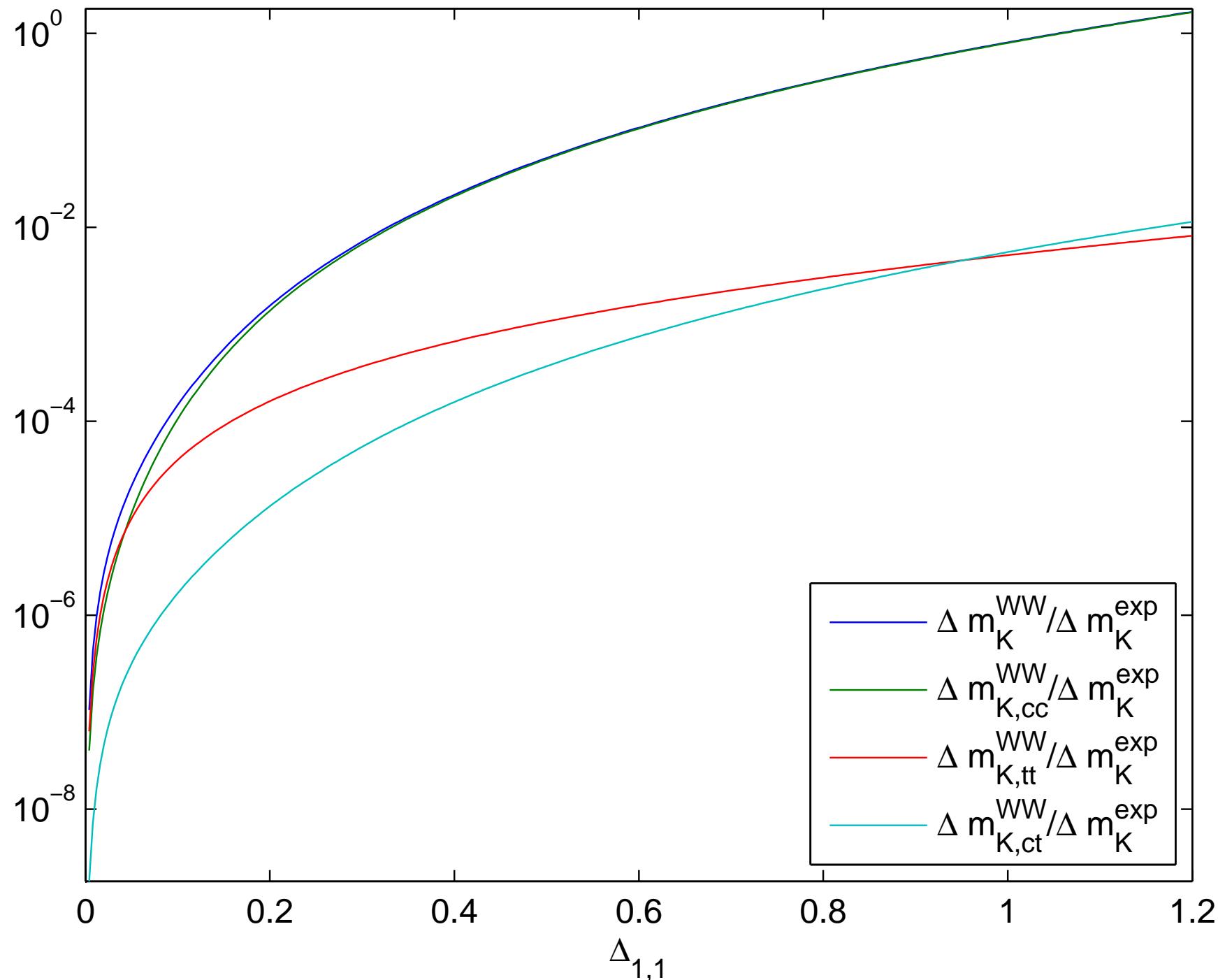
- 构造了含 $W'^{\pm}, Z'$ 的电弱手征拉氏量！
- $W - W'$ 混合是普适的，小的混合并不限制 $W'$ 一定很重！
- 轻子的混合与在标准模型中加入右手中微子的情形相同。
- 夸克的混合涉及右手的**CKM**矩阵。
- 为保证 $W'$ 对 $\Delta m_K$ ,  $|\epsilon_K|$ ,  $\Delta m_{B_d}$ ,  $\Delta m_{B_s}$ 贡献不与实验冲突
  - $M_{W'} \gg M_W$ ;  $g_2 \ll g_1$ ;  $\Delta_{2,1} \ll 1$ ; 特别选择右手CKM矩阵元
  - $W'$ 的额外贡献与 $W$ 和Higgs的贡献相互抵消
- 没单独讨论 $Z'$ : [Y.Zhang, S-Z.Wang, Q.Wang, JHEP03 \(2008\) 047](#)

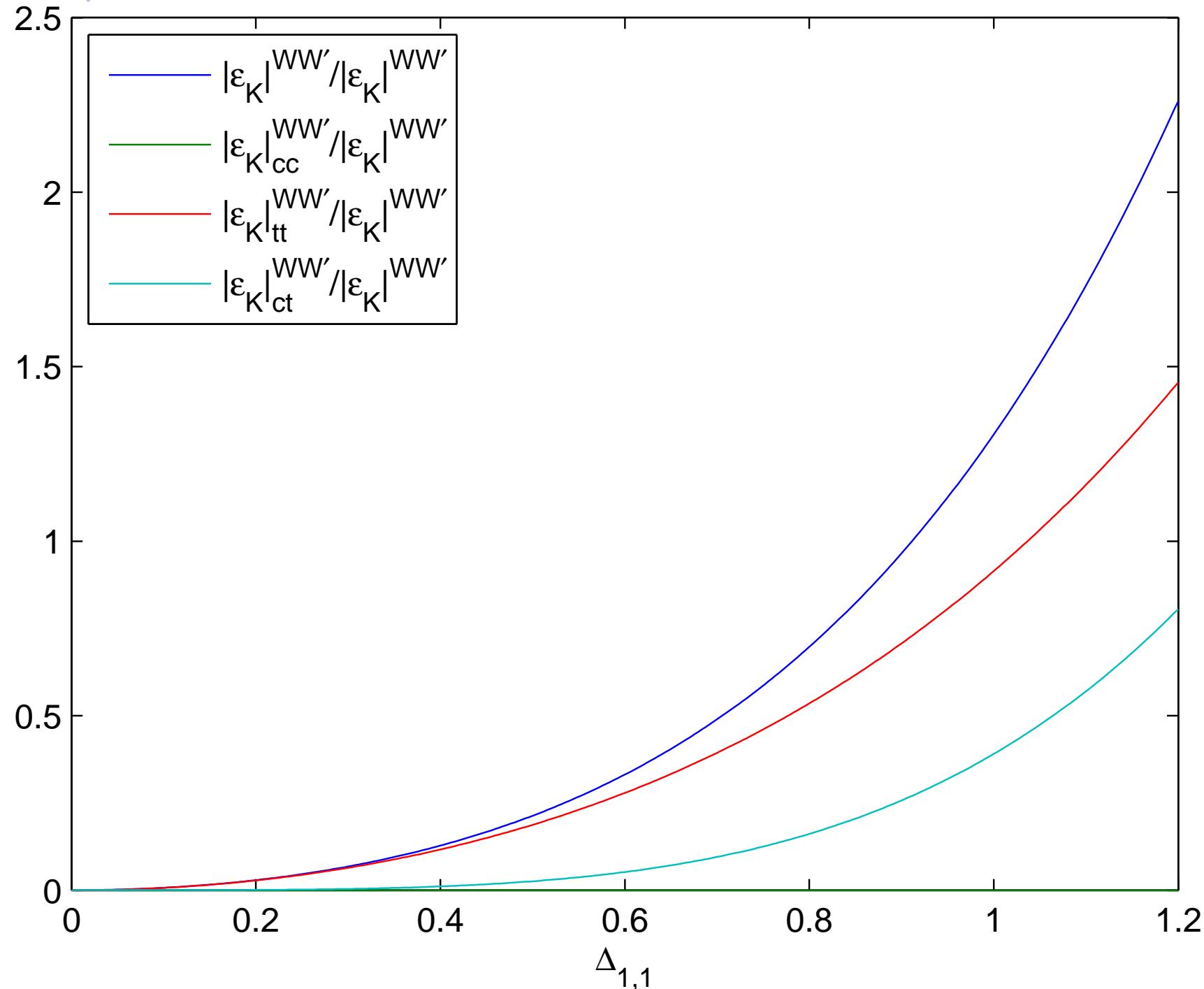


Thanks!

# Backup







$$\mathcal{L}_{\text{CC}}|_{\text{quark}} = -\frac{1}{\sqrt{2}} \bar{q}_\alpha (\tau^+ W_1^+ + \tau^- W_1^-) [(g_L + \Delta_{CL}) P_L + \Delta_{CR} P_R] q_\alpha$$

$$\mathcal{L}_{\text{NC}}|_{\text{quark}} = -\frac{1}{2} \bar{q}_\alpha \not{Z} \left\{ T_{3L} \left[ \frac{g_L}{\cos \theta_W} (1 - \gamma_5) + \Delta_{NVT} + \Delta_{NAT} \gamma_5 \right] + Q \left( \frac{2g_L \sin^2 \theta_W}{\cos \theta_W} + \Delta_{NVQ} + \Delta_{NAQ} \gamma_5 \right) \right\} q_\alpha$$

$$\mathcal{L}_{\text{EM}}|_{\text{quark}} = -\frac{1}{2} \bar{q}_\alpha \not{A} [Q(2e + \Delta_{EVQ} + \Delta_{EAQ} \gamma_5) + T_{3L}(\Delta_{EVT} + \Delta_{EAT} \gamma_5)] q_\alpha$$

$$\Delta_{CL} = g_1 [\cos \zeta (1 - \delta_{L,1} - \delta_{L,4}) - 1] + g_2 \sin \zeta (\delta_{R,2} - \delta_{R,6}) ,$$

$$\Delta_{CR} = g_1 \cos \zeta (\delta_{L,2} - \delta_{L,6}) + g_2 \sin \zeta (1 - \delta_{R,1} - \delta_{R,4})$$

$$\Delta_{NVT} = -\frac{g_1}{\cos \theta_W} + g_1 x_1 [1 - \delta_{L,1} + \delta_{L,4} + \delta_{L,2} + \delta_{L,6} - \frac{2}{Y} (\delta_{L,3} + \delta_{L,7} + \delta_{L,5})] + g_R y_1 [1 - \delta_{R,1}$$

$$+ \delta_{R,4} + \delta_{R,2} + \delta_{R,6} - \frac{2}{Y} (\delta_{R,3} + \delta_{R,7} + \delta_{R,5})] + g v_1 [-2 + \delta_{L,1} + \delta_{R,1} - \delta_{R,2} - \delta_{L,2} - \delta_{L,4}$$

$$- \delta_{R,4} - \delta_{R,6} - \delta_{L,6} + 2Y \delta_{L,7} + 2Y \delta_{R,7} + \frac{2}{Y} (\delta_{L,3} + \delta_{R,5} + \delta_{R,3} + \delta_{L,5})] ,$$

$$\Delta_{NAT} = \frac{g_1}{\cos \theta_W} - g_1 x_1 [1 - \delta_{L,1} + \delta_{L,4} - \delta_{L,2} - \delta_{L,6} - \frac{2}{Y} (\delta_{L,3} + \delta_{L,7} - \delta_{L,5})] - g_2 y_1 [1 - \delta_{R,1}$$

$$+ \delta_{R,4} - \delta_{R,2} - \delta_{R,6} + \frac{2}{Y} (\delta_{R,3} + \delta_{R,7} - \delta_{R,5})] + g v_1 [\delta_{L,1} - \delta_{R,1} - \delta_{R,2} + \delta_{L,2} - \delta_{L,4} + \delta_{R,4}$$

$$- \delta_{R,6} + \delta_{L,6} + 2Y \delta_{L,7} - 2Y \delta_{R,7} + \frac{2}{Y} (-\delta_{L,3} - \delta_{R,5} + \delta_{R,3} + \delta_{L,5})] ,$$



$$\Delta_{NVQ} = \frac{2g_1 \sin^2 \theta_W}{\cos \theta_W} + \frac{2}{Y} [g_1 x_1 (\delta_{L,3} + \delta_{L,7} + \delta_{L,5}) + g_2 y_1 (\delta_{R,3} + \delta_{R,7} + \delta_{R,5}) + gv_1 (Y - \delta_{L,3} - \delta_{R,5} - \delta_{R,3} - \delta_{L,5})] ,$$

$$\Delta_{NAQ} = -\frac{2}{Y} [g_1 x_1 (\delta_{L,3} + \delta_{L,7} - \delta_{L,5}) - g_2 y_1 (\delta_{R,3} + \delta_{R,7} - \delta_{R,5}) + gv_1 (-\delta_{L,3} - \delta_{R,5} + \delta_{R,3} + \delta_{L,5})] ,$$

$$\Delta_{EVQ} = -2e + \frac{2}{Y} [g_1 x_3 (\delta_{L,3} + \delta_{L,7} + \delta_{L,5}) + g_2 y_3 (\delta_{R,3} + \delta_{R,7} + \delta_{R,5}) + gv_3 (Y - \delta_{L,3} - \delta_{R,5} - \delta_{R,3} - \delta_{L,5})] ,$$

$$\Delta_{EAQ} = -\frac{2}{Y} [g_1 x_3 (\delta_{L,3} + \delta_{L,7} - \delta_{L,5}) - g_2 y_3 (\delta_{R,3} + \delta_{R,7} - \delta_{R,5}) + gv_3 (-\delta_{L,3} - \delta_{R,5} + \delta_{R,3} + \delta_{L,5})]$$

$$\begin{aligned} \Delta_{EVT} = & g_1 x_3 [1 - \delta_{L,1} + \delta_{L,4} + \delta_{L,2} + \delta_{L,6} - \frac{2}{Y} (\delta_{L,3} + \delta_{L,7} + \delta_{L,5})] + g_2 y_3 [1 - \delta_{R,1} + \delta_{R,4} + \delta_{R,2} \\ & + \delta_{R,6} - \frac{2}{Y} ((\delta_{R,3} + \delta_{R,7} + \delta_{R,5}))] + gv_3 [-2 + \delta_{L,1} + \delta_{R,1} - \delta_{R,2} - \delta_{L,2} - \delta_{L,4} - \delta_{R,4} - \delta_{R,6} \\ & - \delta_{L,6} + 2Y \delta_{L,7} + 2Y \delta_{R,7} + \frac{2}{Y} (\delta_{L,3} + \delta_{R,5} + \delta_{R,3} + \delta_{L,5})] \end{aligned}$$

$$\begin{aligned} \Delta_{EAT} = & -g_1 x_3 [1 - \delta_{L,1} + \delta_{L,4} - \delta_{L,2} - \delta_{L,6} - \frac{2}{Y} (\delta_{L,3} + \delta_{L,7} - \delta_{L,5})] - g_2 y_3 [1 - \delta_{R,1} + \delta_{R,4} - \delta_{R,2} \\ & - \delta_{R,6} + \frac{2}{Y} ((\delta_{R,3} + \delta_{R,7} - \delta_{R,5}))] + gv_3 [\delta_{L,1} - \delta_{R,1} - \delta_{R,2} + \delta_{L,2} - \delta_{L,4} + \delta_{R,4} - \delta_{R,6} \\ & + \delta_{L,6} + 2Y \delta_{L,7} - 2Y \delta_{R,7} + \frac{2}{Y} (-\delta_{L,3} - \delta_{R,5} + \delta_{R,3} + \delta_{L,5})] . \end{aligned}$$